



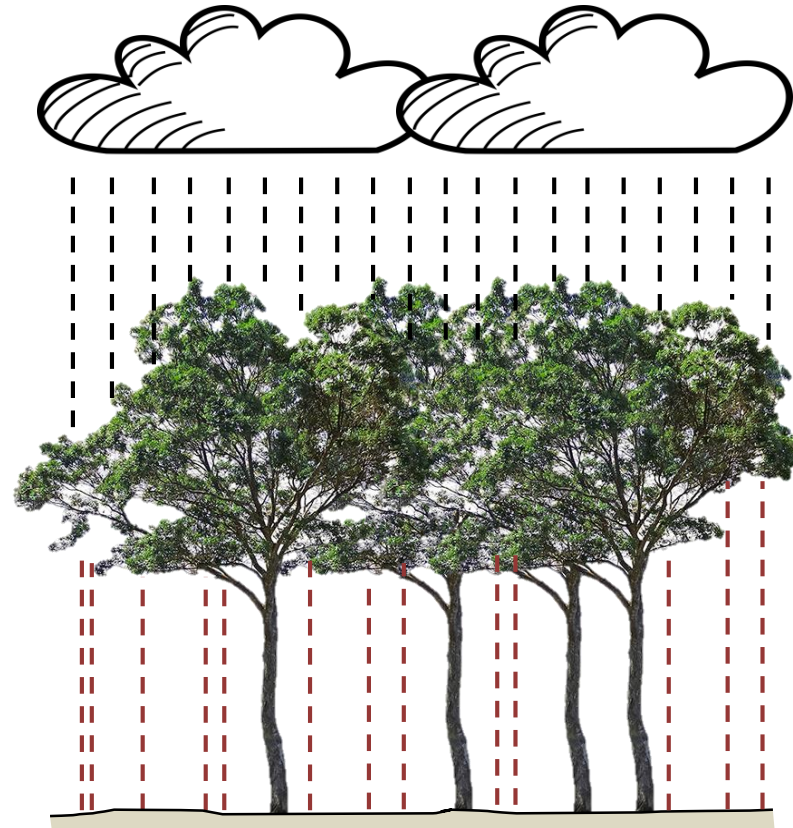
# IMPACTO DA ESCOLHA DA FUNÇÃO DE VEROSSIMILHANÇA NA ESTIMATIVA DE INCERTEZA E INFERÊNCIA DOS PARÂMETROS DE UM MODELO DE INTERCEPTAÇÃO



Debora Yumi de Oliveira, Pedro Luiz Borges Chaffe,  
João Henrique Macedo Sá



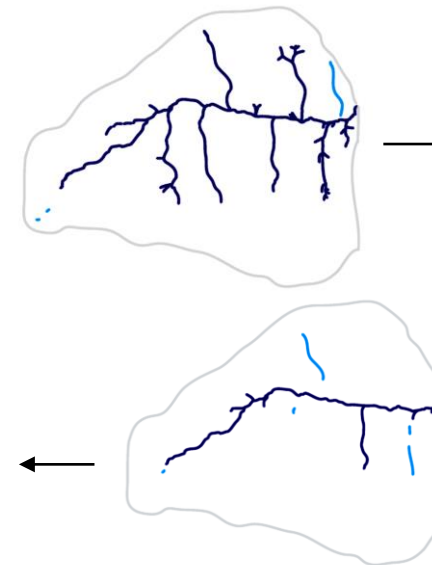
Florianópolis, 27 de novembro de 2017



INTERCEPTAÇÃO

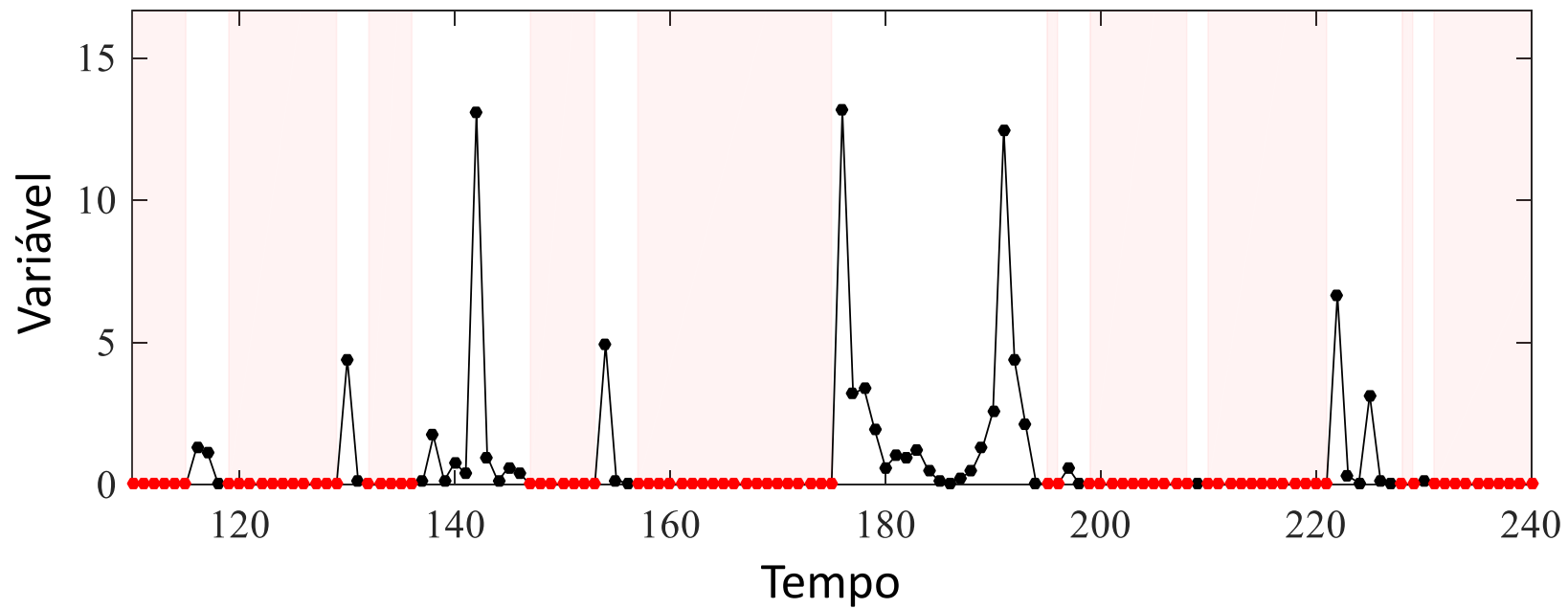
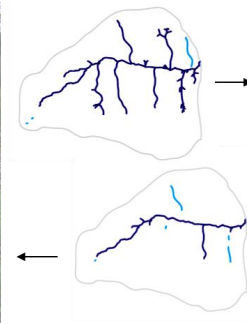
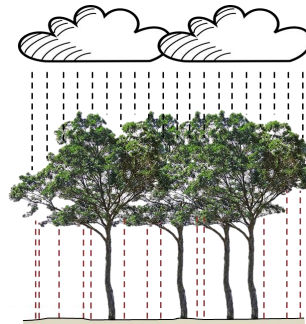


CONCENTRAÇÃO DE SUBSTÂNCIAS NA ÁGUA

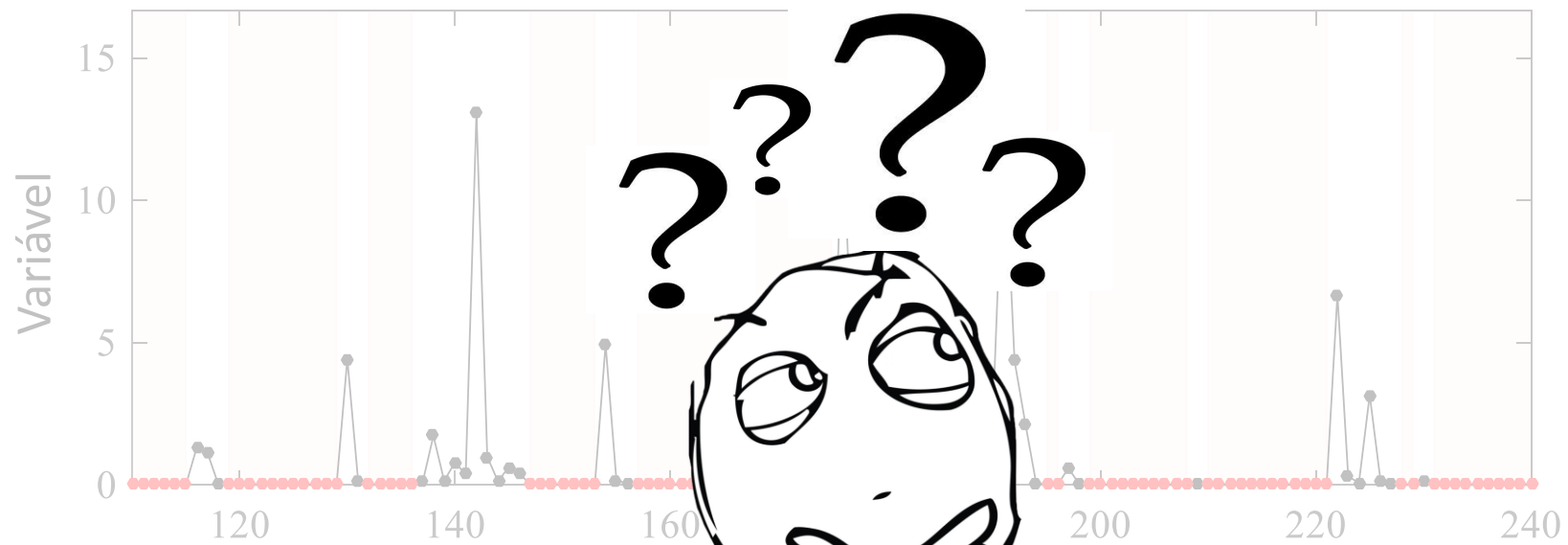
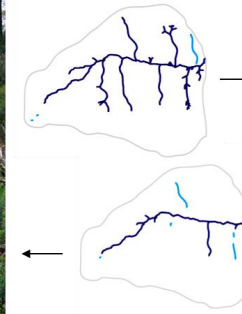
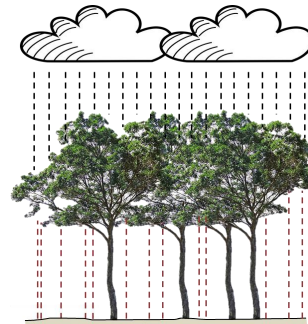


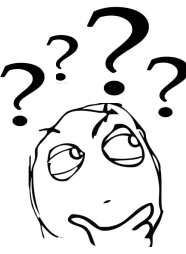
VAZÃO EM RIOS INTERMITENTES

Série de dados possui muitas **observações iguais a zero!**



# E qual o problema???



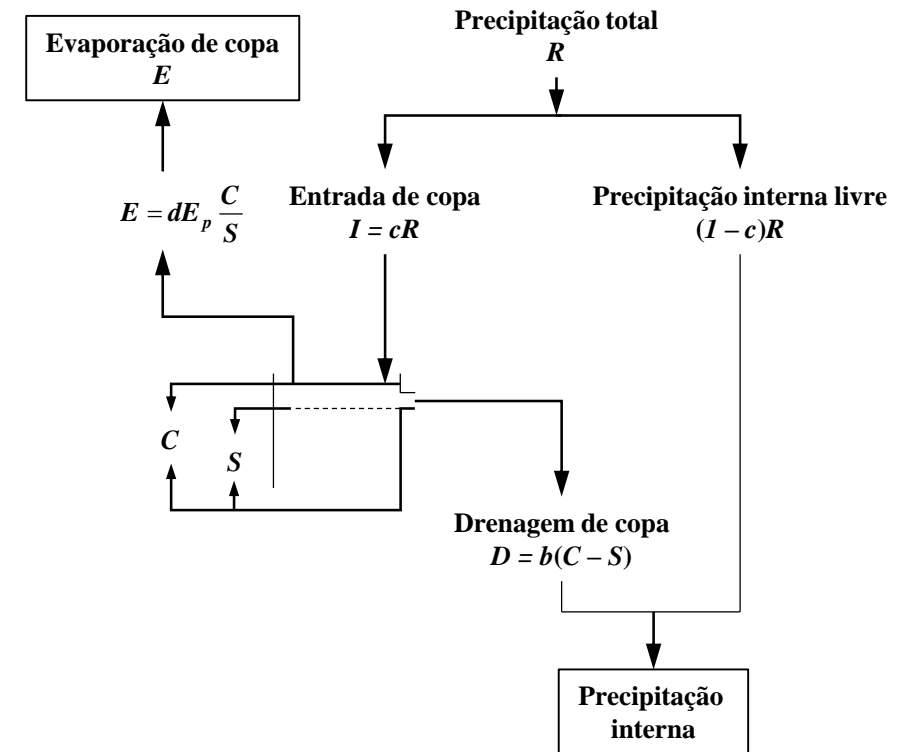
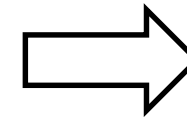


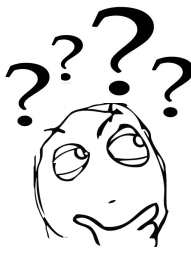
# COMPREENSÃO DOS PROCESSOS HIDROLÓGICOS

## MONITORAMENTO



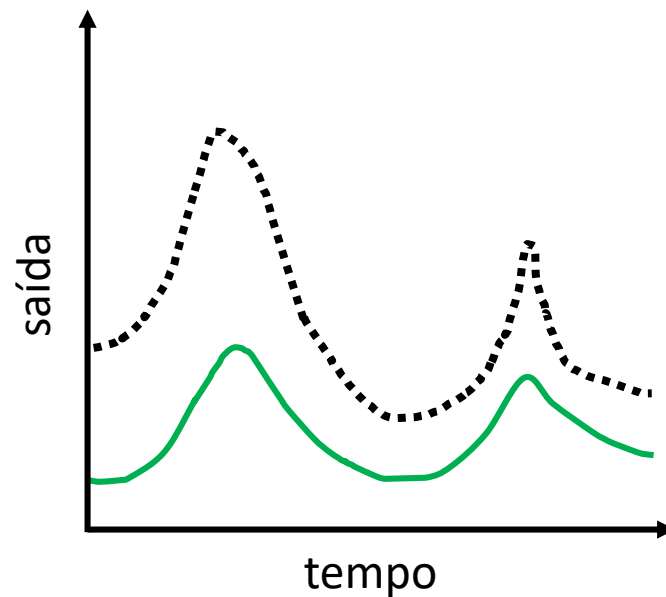
## MODELAGEM



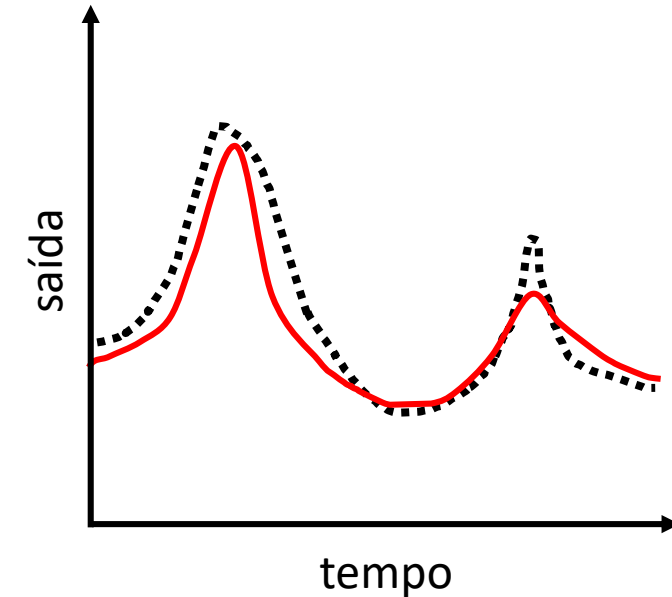
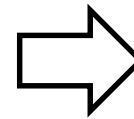


## CALIBRAÇÃO DE MODELOS

- Todo modelo, por mais realista que seja, possui parâmetros que precisam ser ajustados

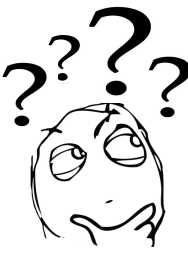


calibração



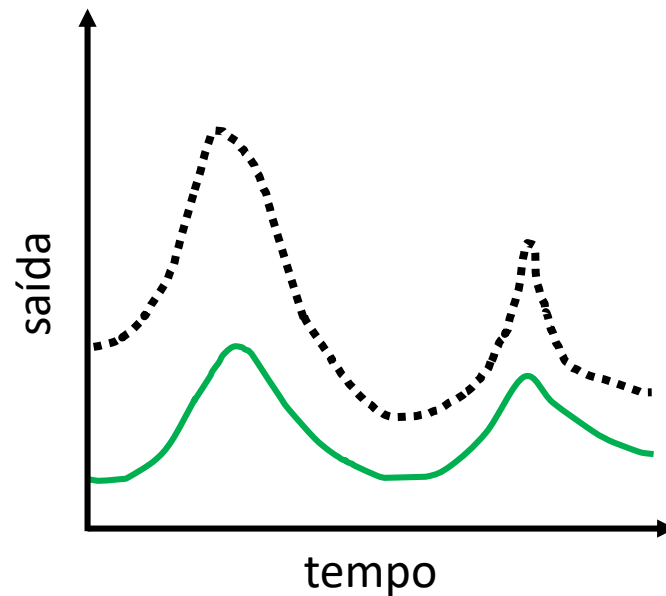
..... Observado      — Simulado

..... Observado      — Simulado

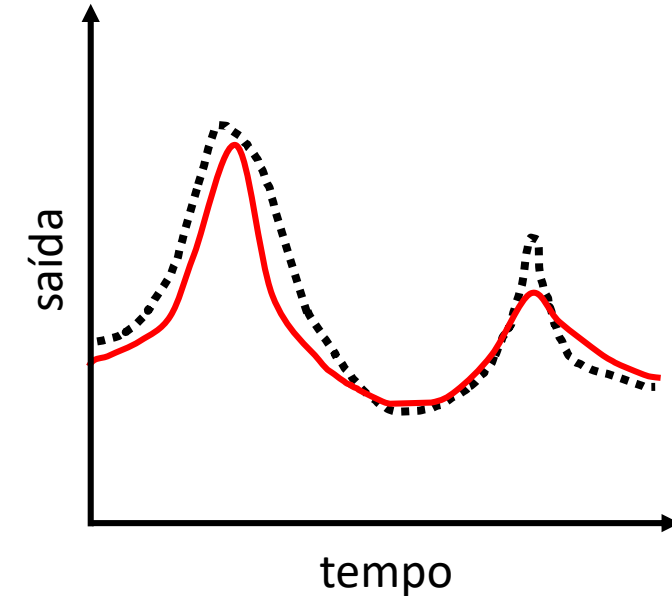
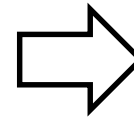


## CALIBRAÇÃO TRADICIONAL

- Objetivo é encontrar um conjunto de parâmetros que minimize a distância entre a saída do modelo e as observações

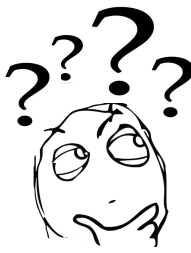


calibração



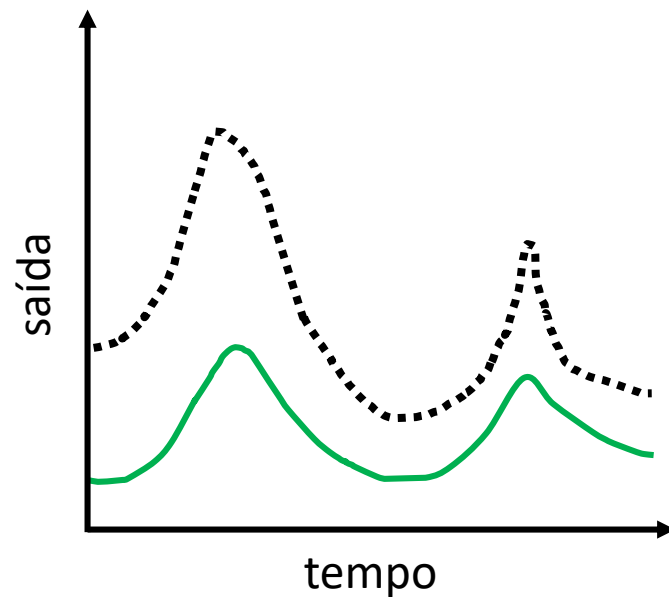
..... Observado      — Simulado

..... Observado      — Simulado

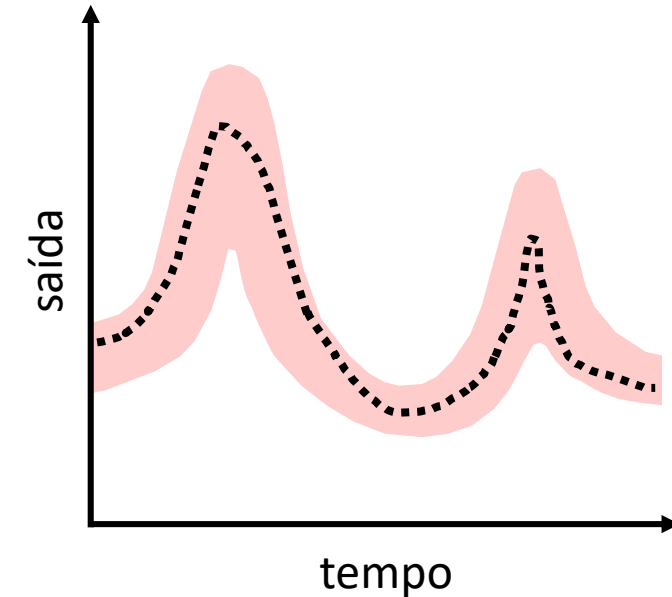
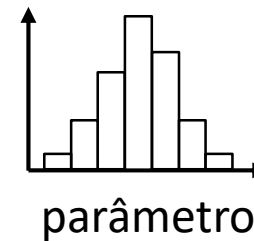
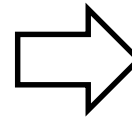


## CALIBRAÇÃO E ANÁLISE DE INCERTEZA

- Diversas fontes de erro envolvidas no processo de modelagem: inferência dos parâmetros deve ser acompanhada da **estimativa de incerteza**



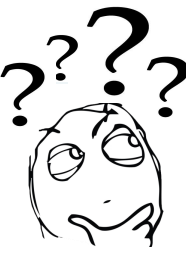
calibração



..... Observado      — Simulado

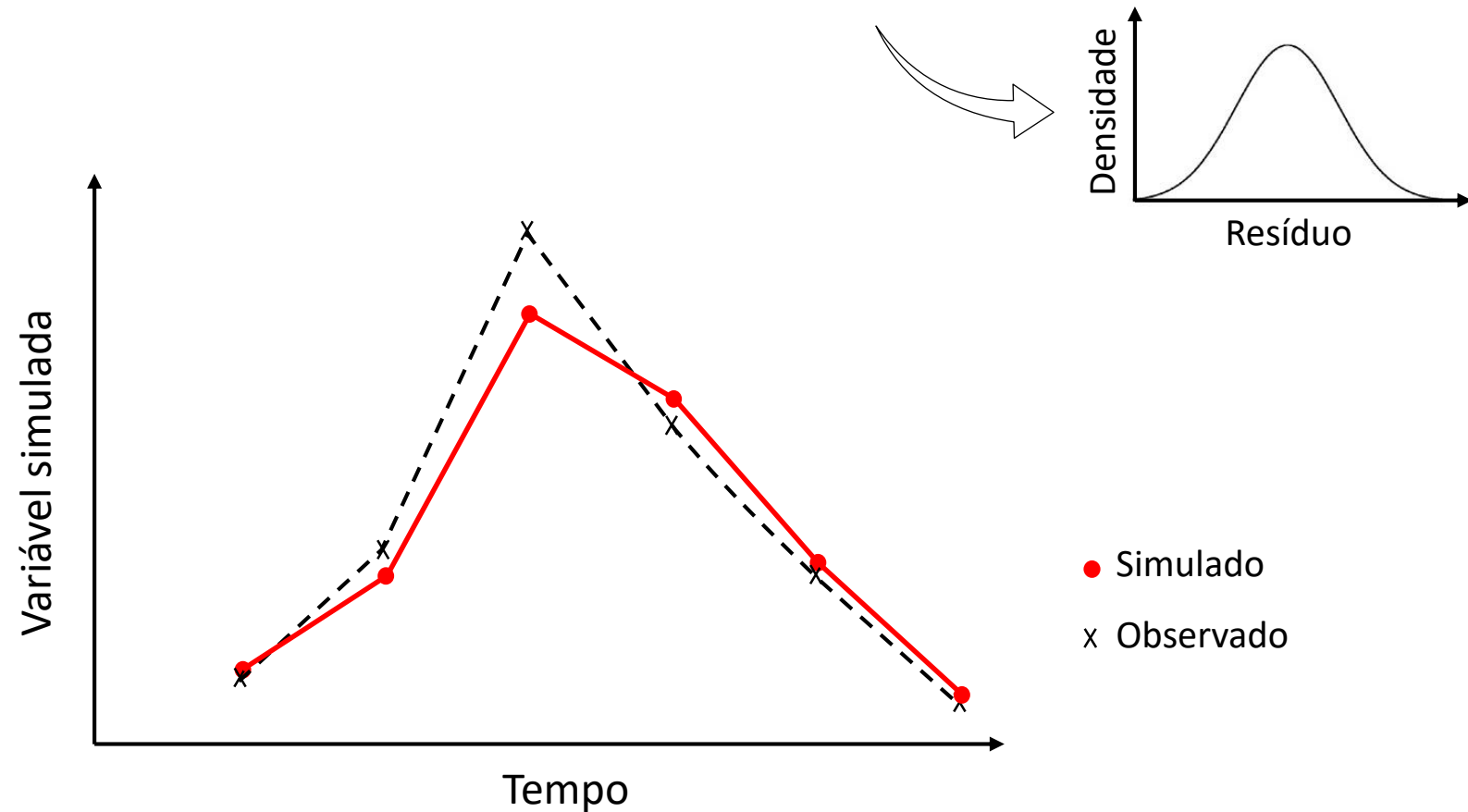
..... Observado      ■ Incerteza

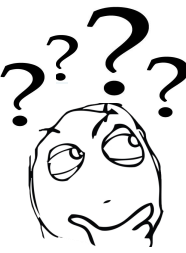




# CALIBRAÇÃO E ANÁLISE DE INCERTEZA

$$\text{OBSERVADO} = \text{SIMULADO} + \text{RESÍDUO}$$



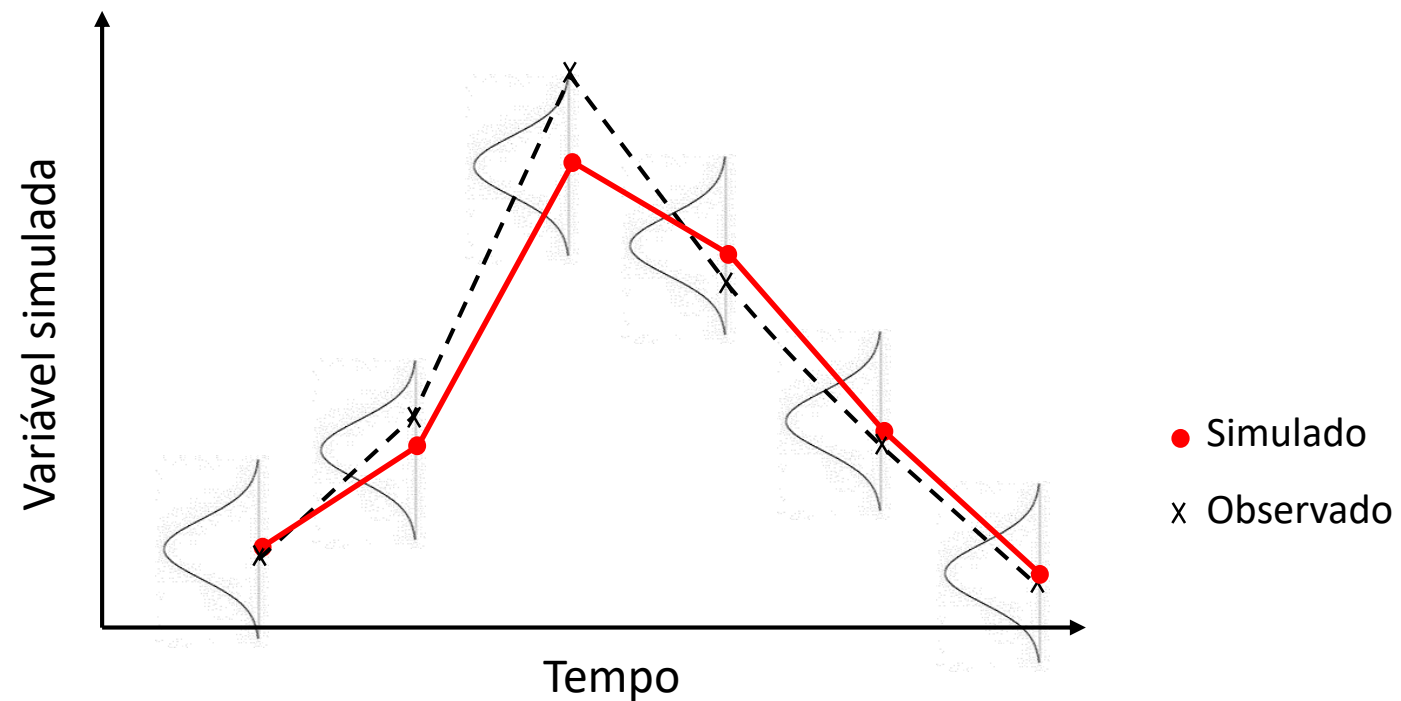


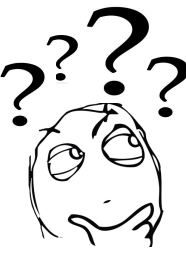
# CALIBRAÇÃO E ANÁLISE DE INCERTEZA

$$\text{OBSERVADO} = \text{SIMULADO} + \text{RESÍDUO}$$

componente  
determinístico

componente  
probabilístico



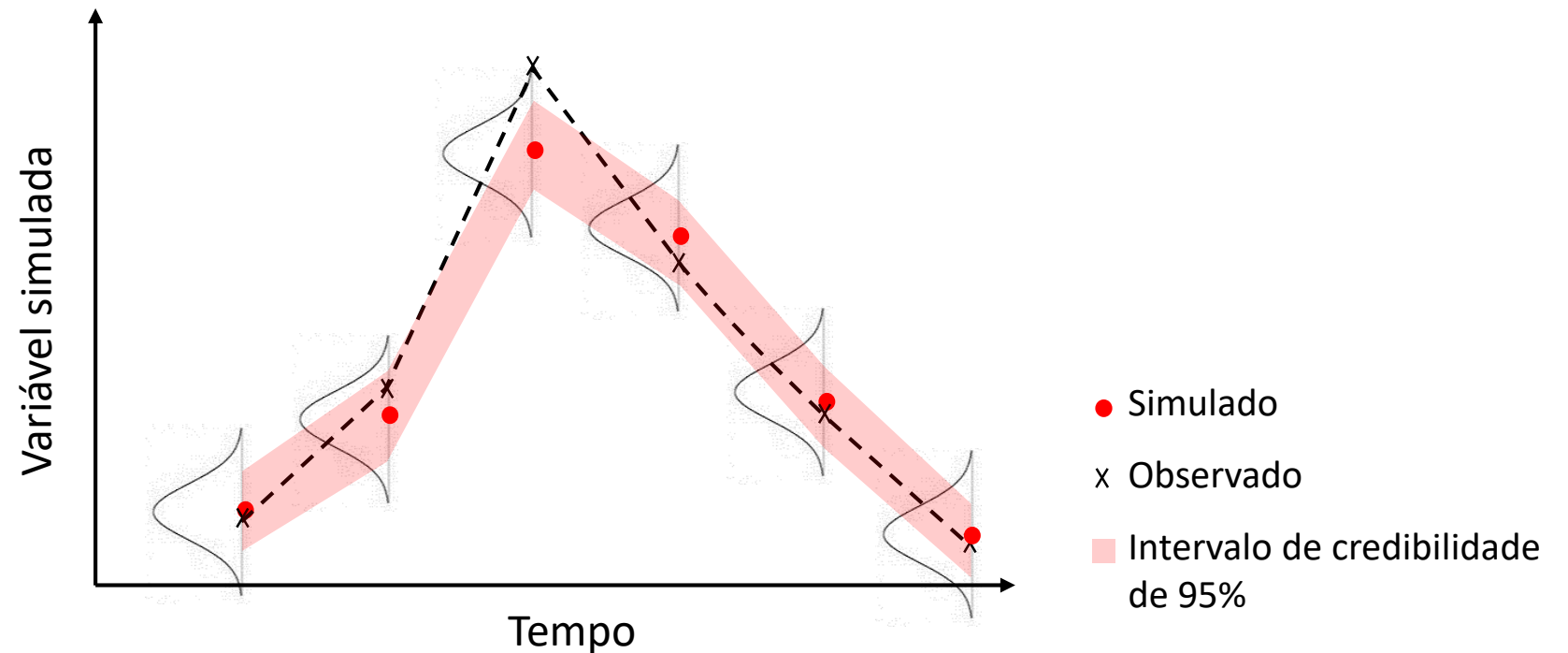


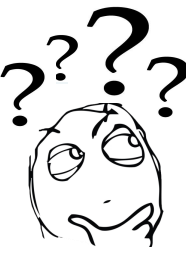
# CALIBRAÇÃO E ANÁLISE DE INCERTEZA

$$\text{OBSERVADO} = \text{SIMULADO} + \text{RESÍDUO}$$

componente  
determinístico

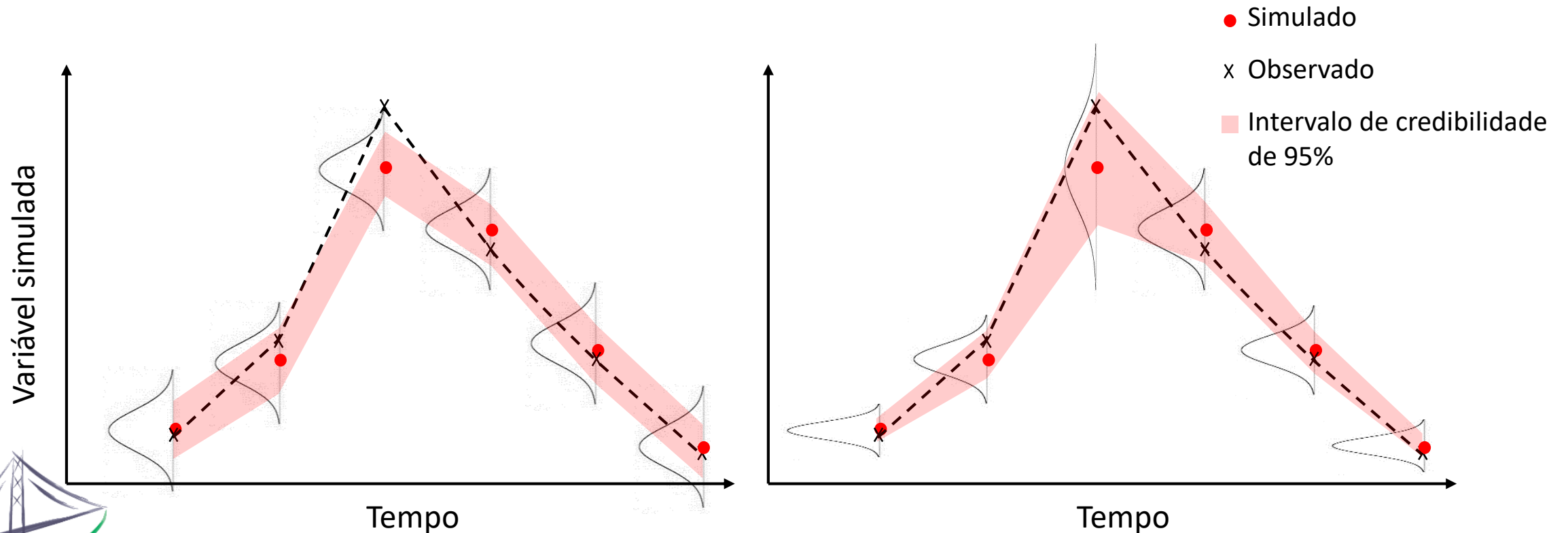
componente  
probabilístico

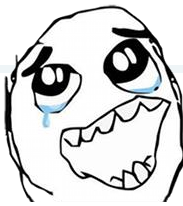




## ANÁLISE DE INCERTEZA

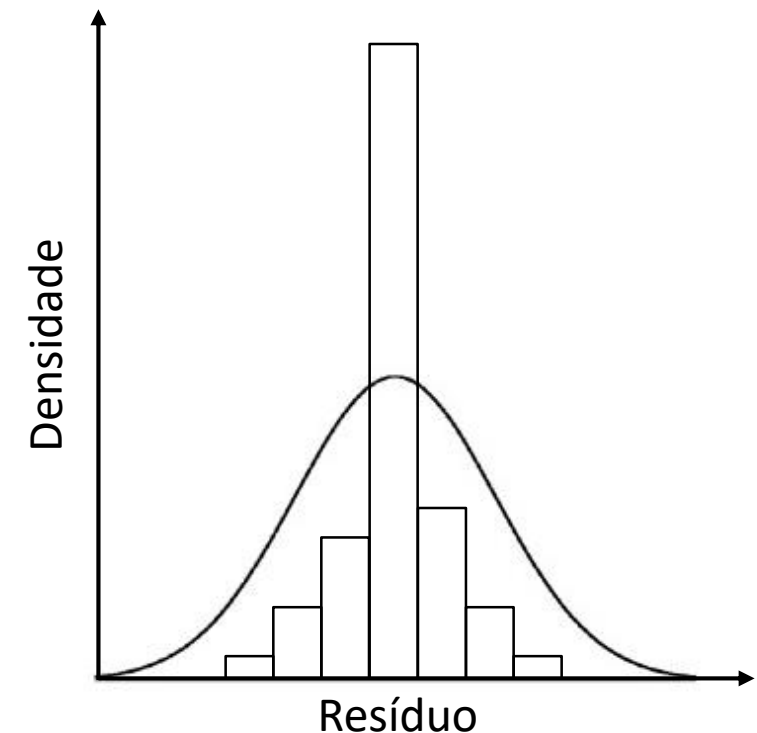
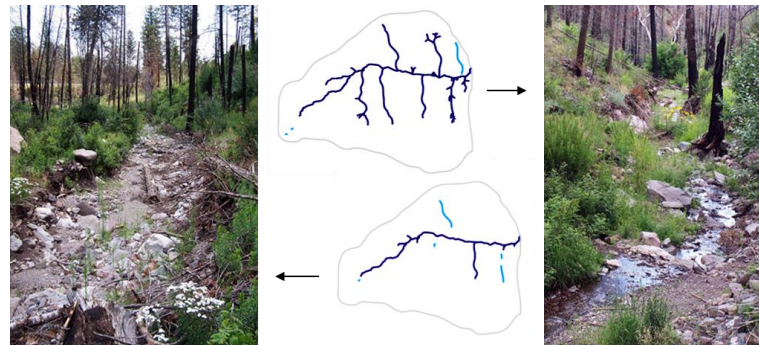
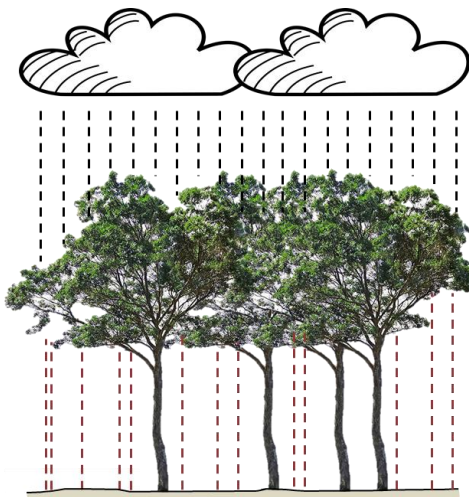
- A qualidade da faixa de incerteza depende da **correta caracterização dos resíduos do modelo**.





## INFLAÇÃO DA DISTRIBUIÇÃO DOS RESÍDUOS COM ZEROS

- Períodos prolongados de observações iguais a zero resultam em uma **distribuição dos resíduos com a predominância de zeros**



## OBJETIVO

Avaliar o impacto da utilização de diferentes modelos para os resíduos na **estimativa dos parâmetros** de um modelo de interceptação assim como na **quantificação da incerteza** associada aos resultados das simulações.



# MATERIAIS E MÉTODOS









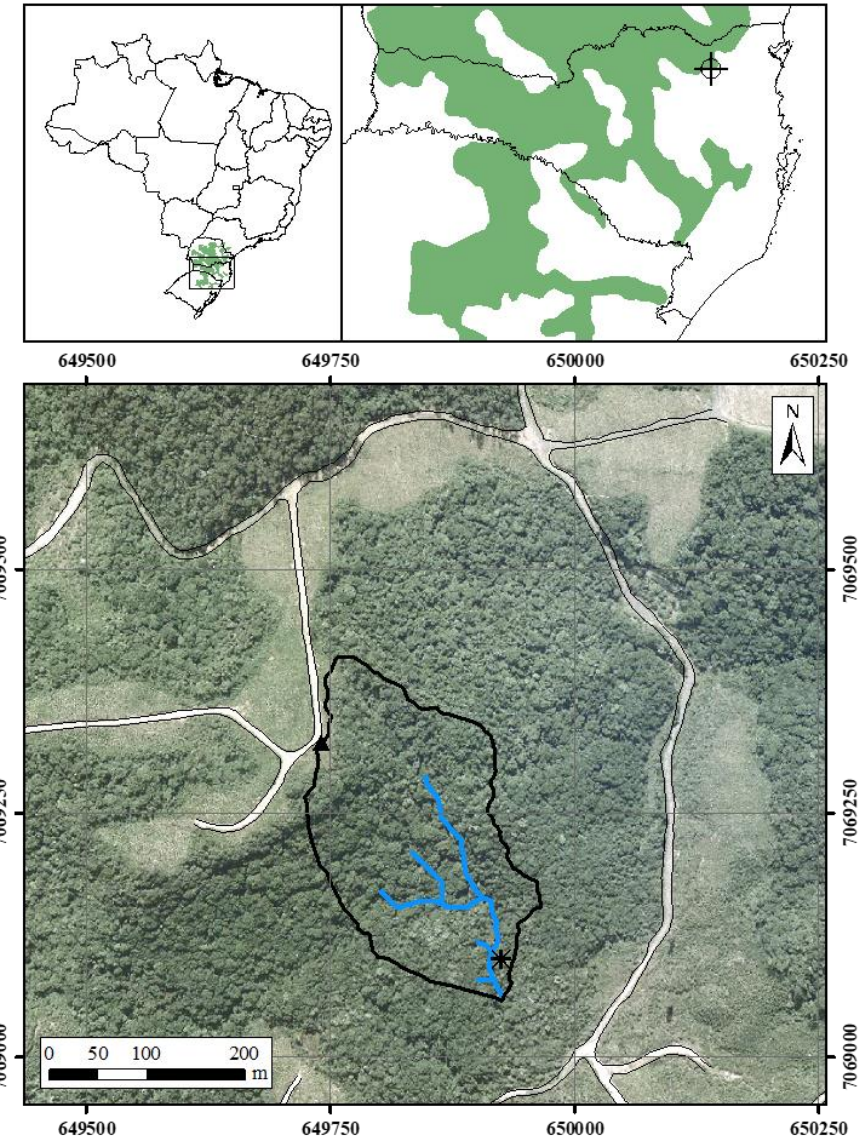
## ÁREA DE ESTUDO

- Monitoramento automático
  - Chuva
  - Chuva interna
- Floresta Ombrófila Mista
- 59 eventos (26/fev a 06/out/2014)

Projeção: UTM Fuso: 22 S  
Datum: SAD 69

### Legenda

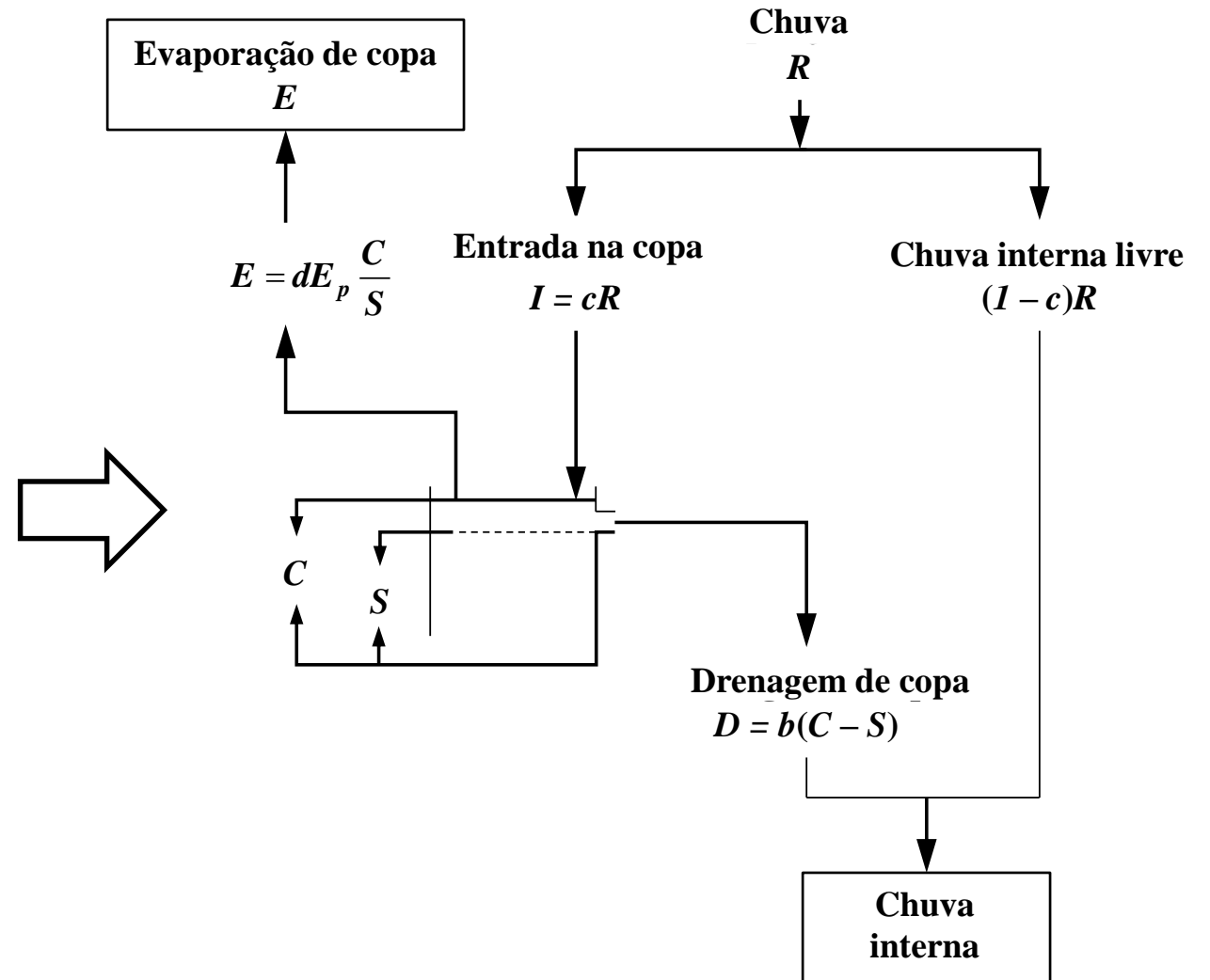
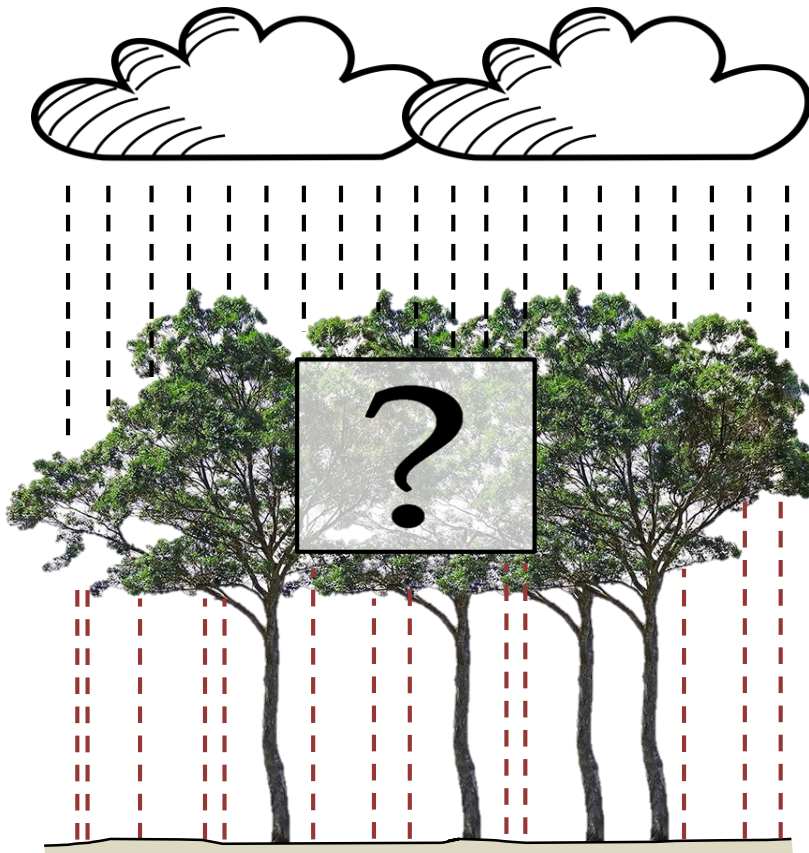
- |   |                          |   |                                 |
|---|--------------------------|---|---------------------------------|
|  | Floresta Ombrófila Mista |  | Medição de precipitação total   |
|  | Bacia do rio Araçuaia    |  | Medição de precipitação interna |
|  | Estradas                 |   |                                 |
|  | Rede de drenagem         |   |                                 |



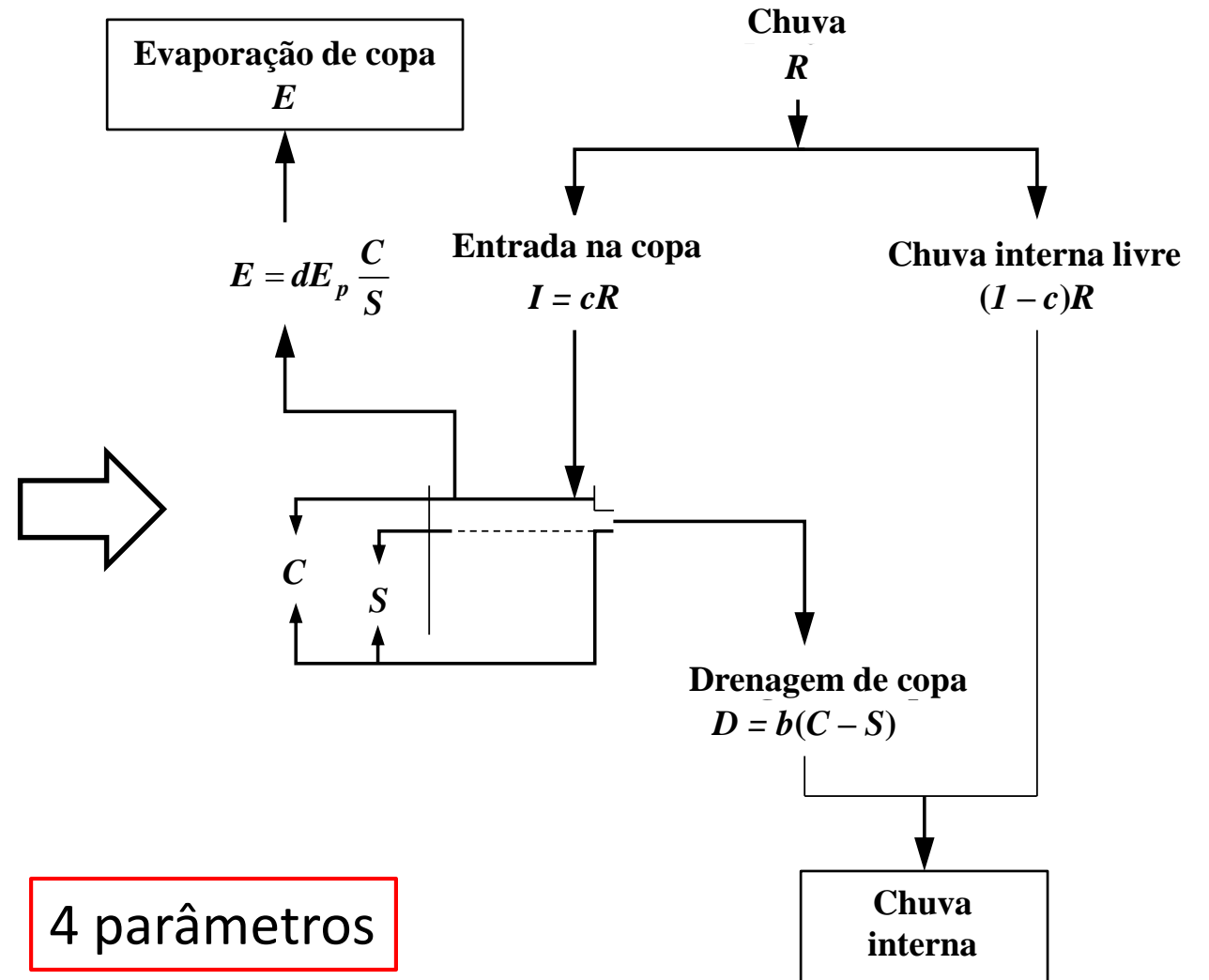
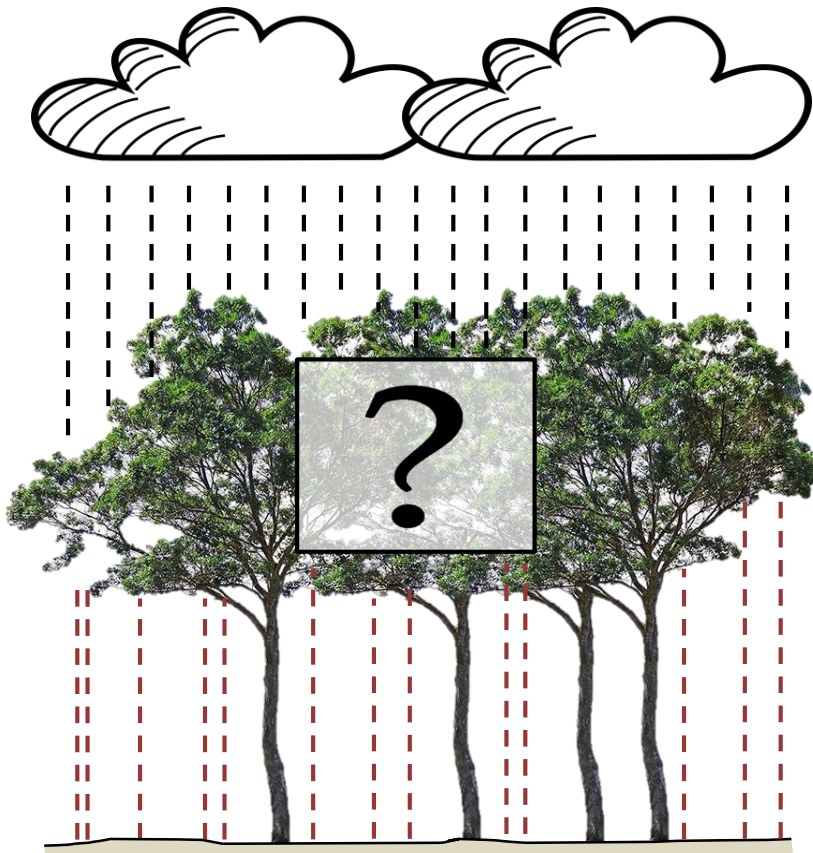
Adaptado de Giglio (2013)



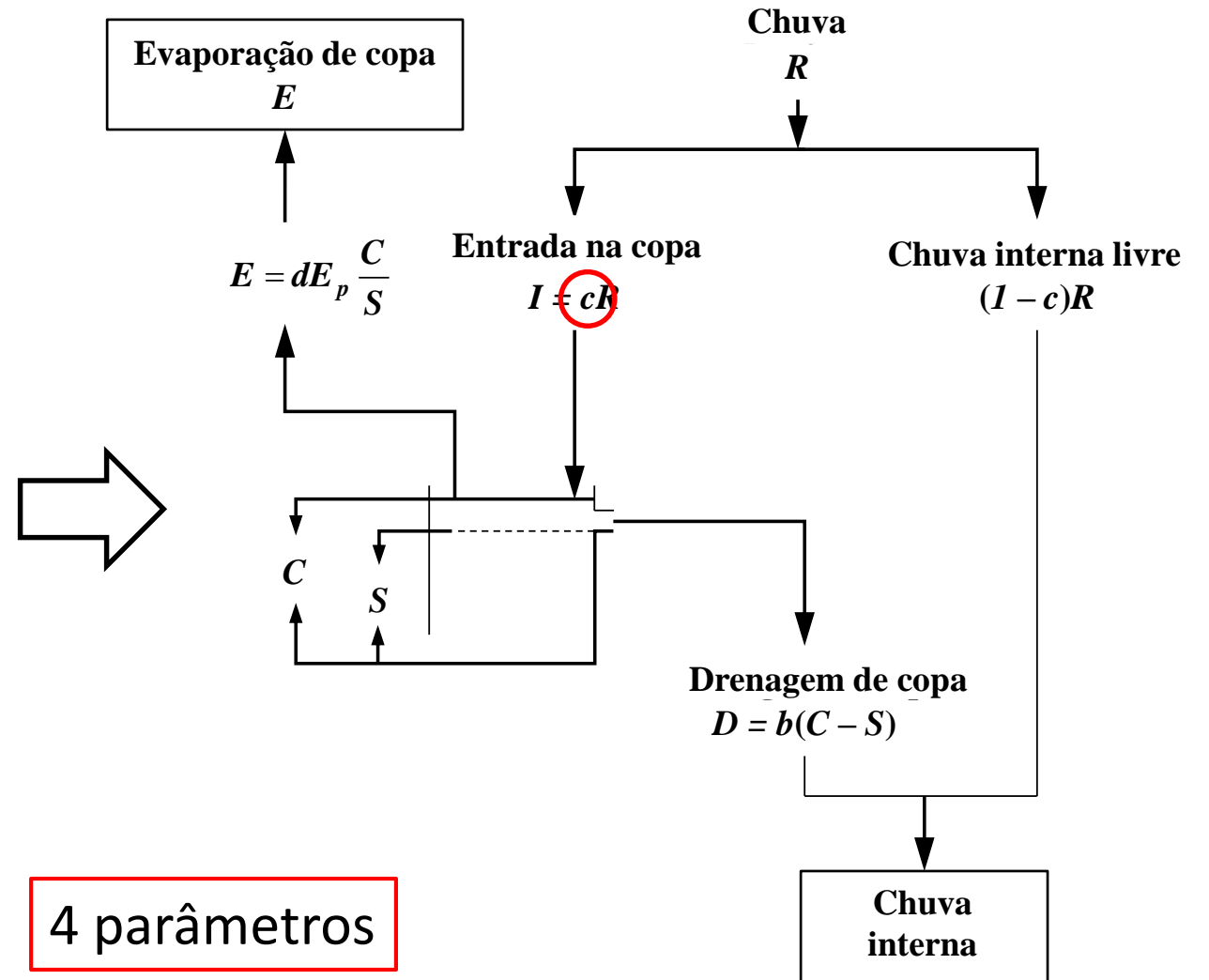
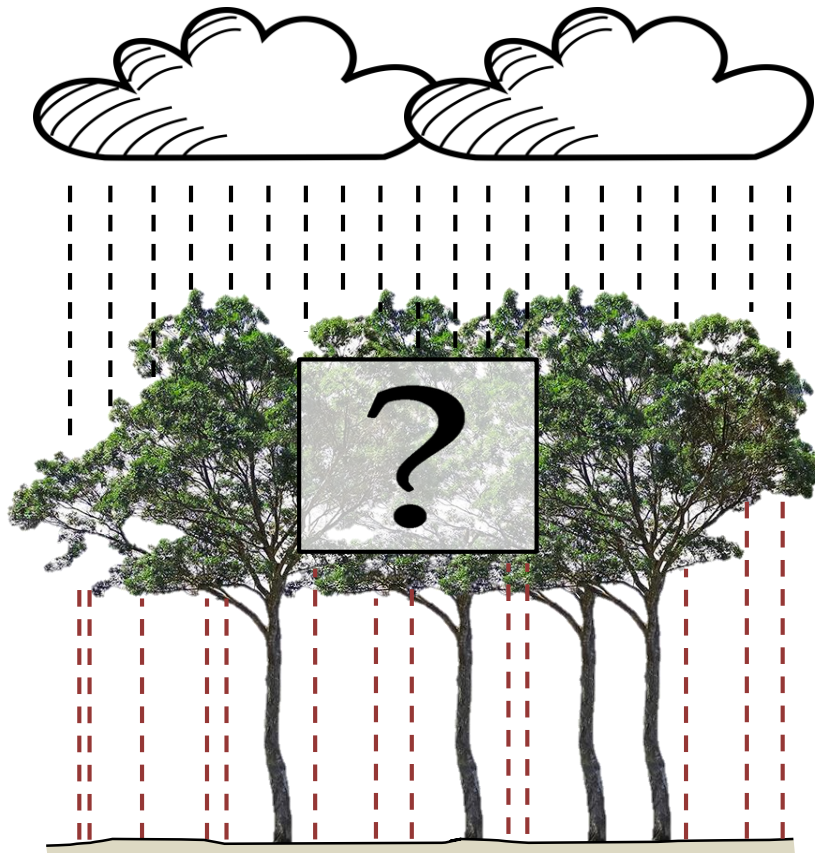
## MODELO DE INTERCEPTAÇÃO (RUTTER MODIFICADO)



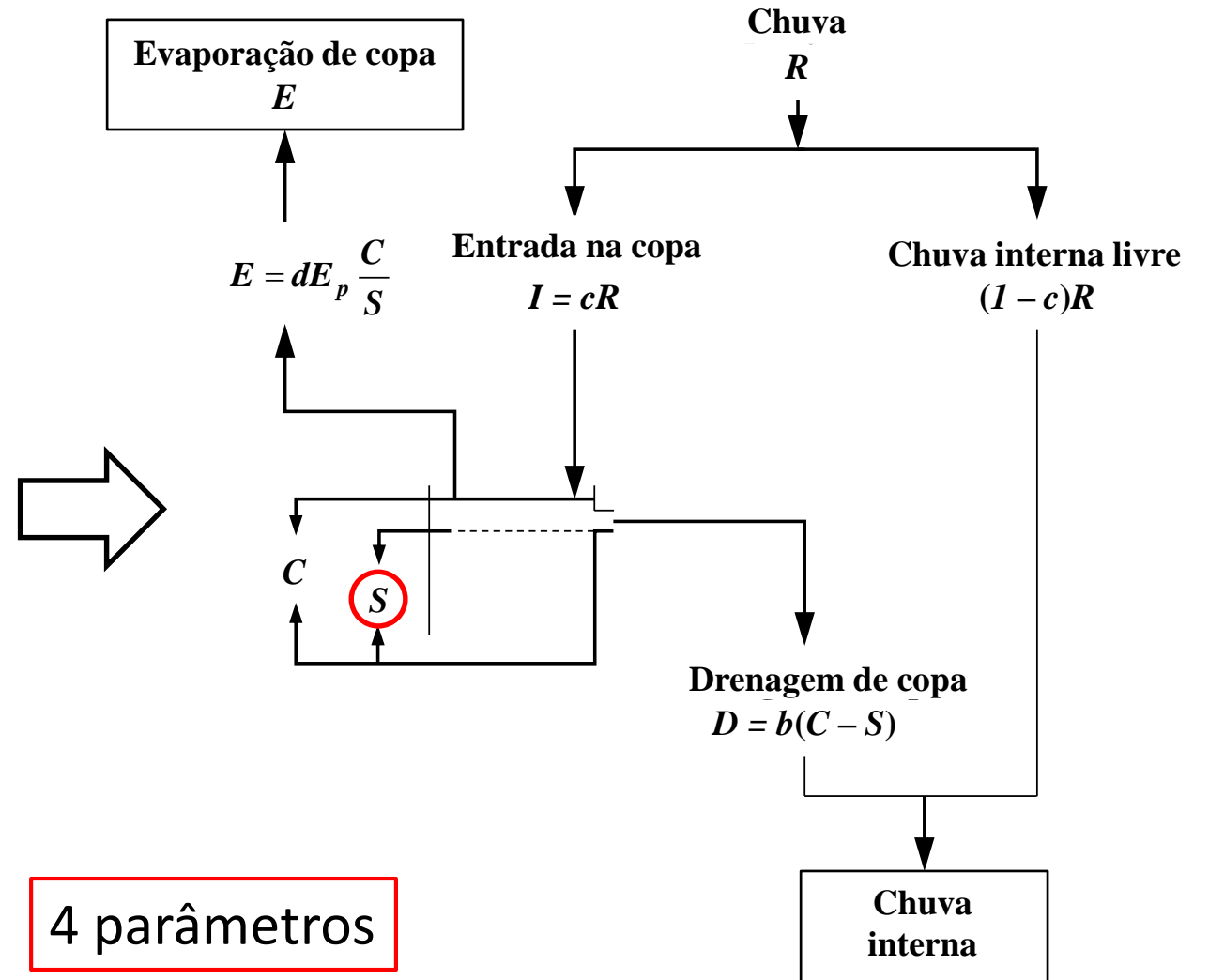
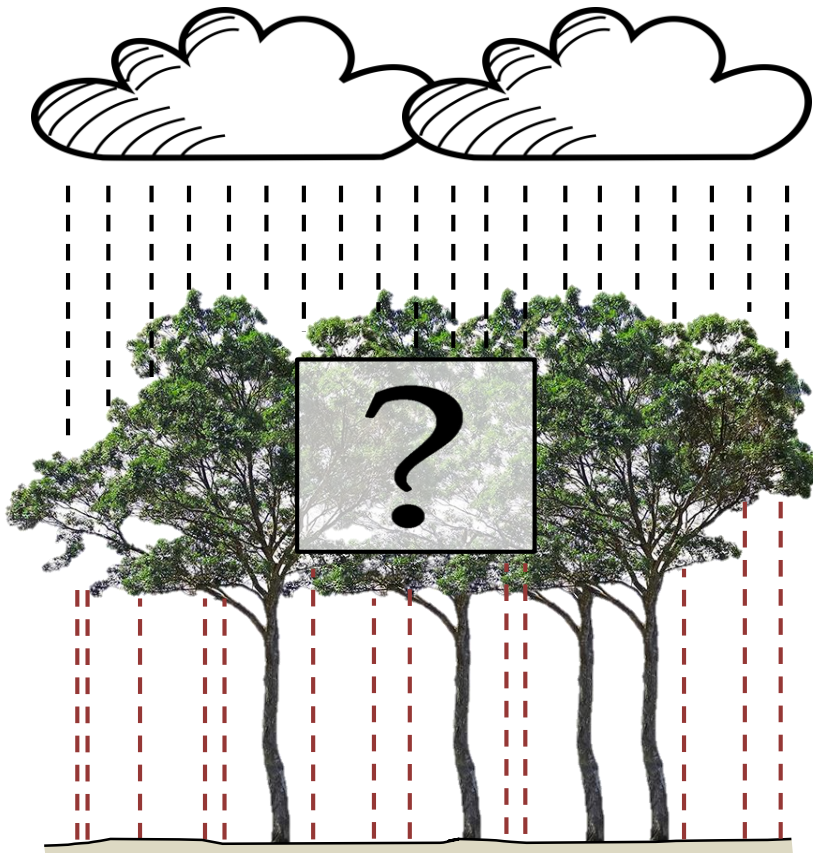
# MODELO DE INTERCEPTAÇÃO (RUTTER MODIFICADO)



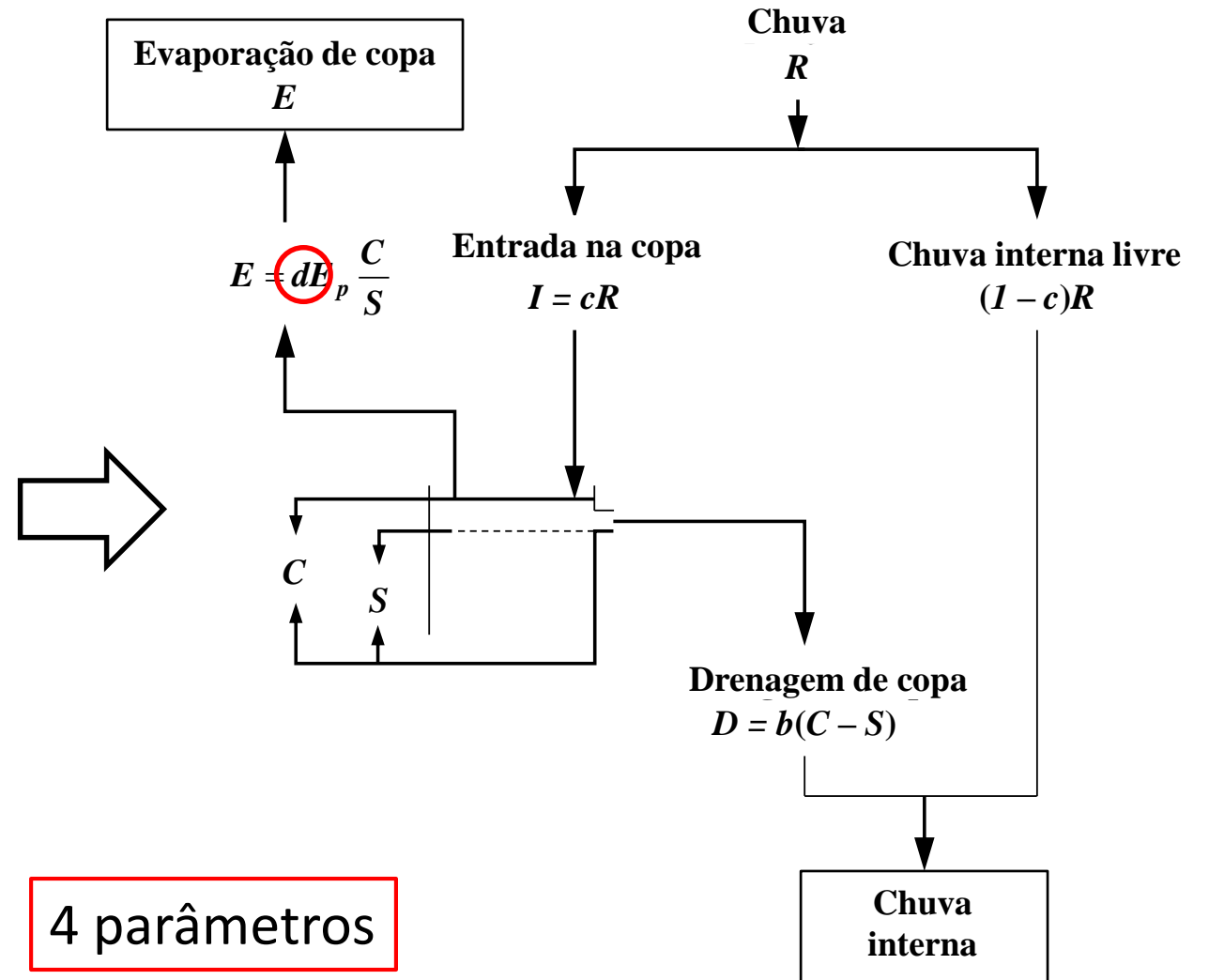
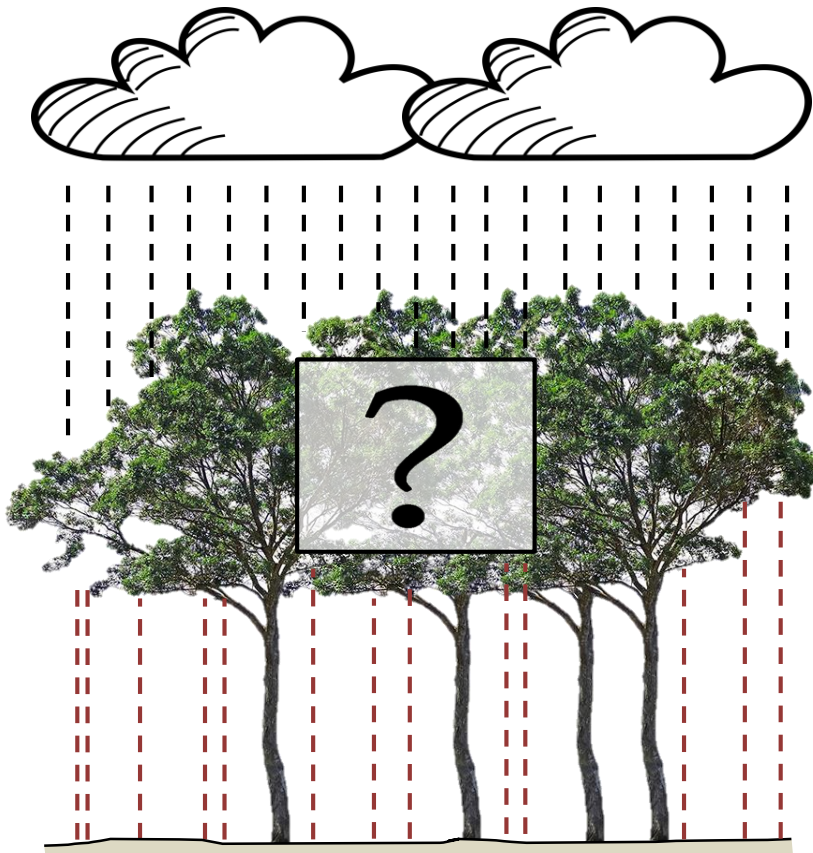
# MODELO DE INTERCEPTAÇÃO (RUTTER MODIFICADO)



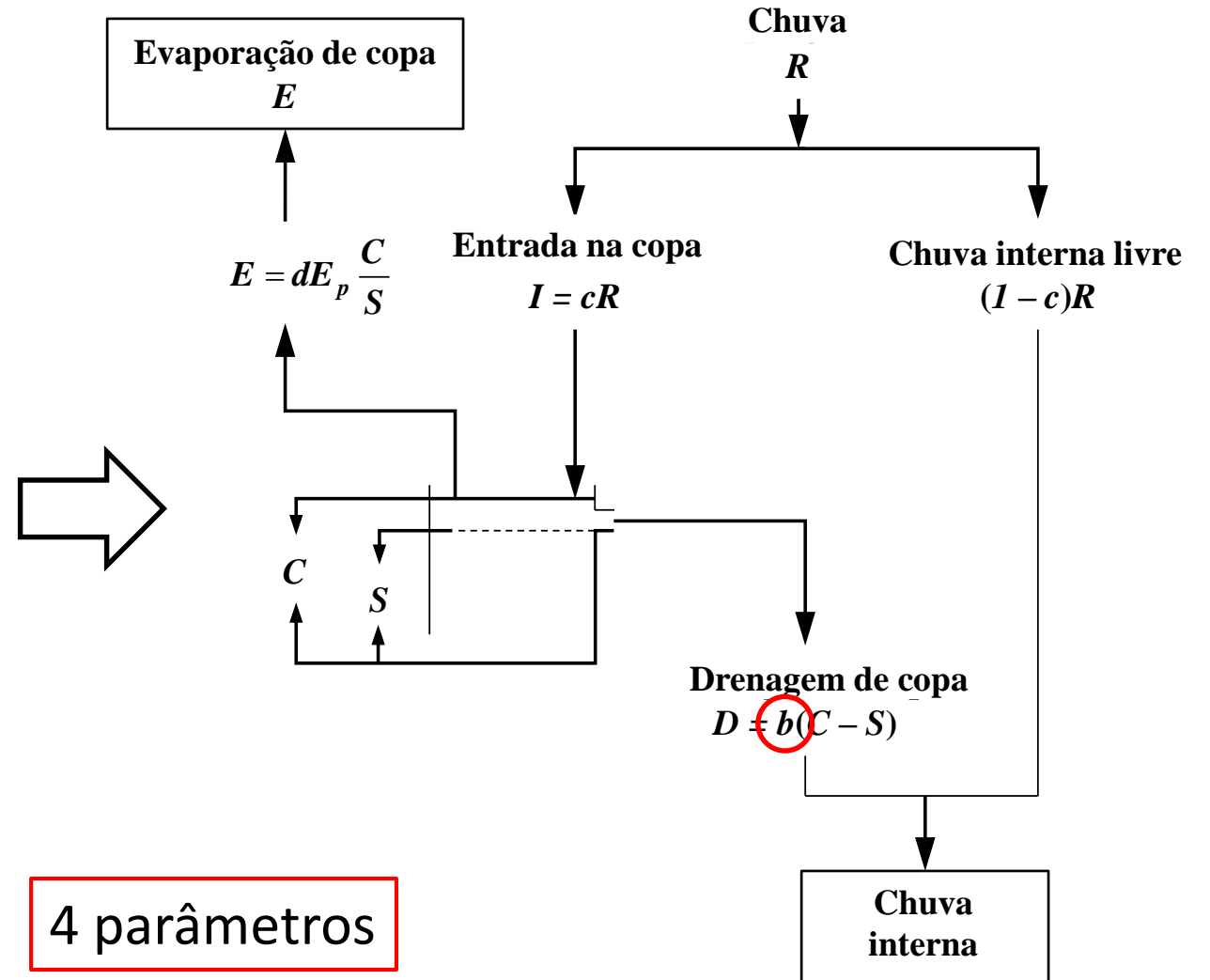
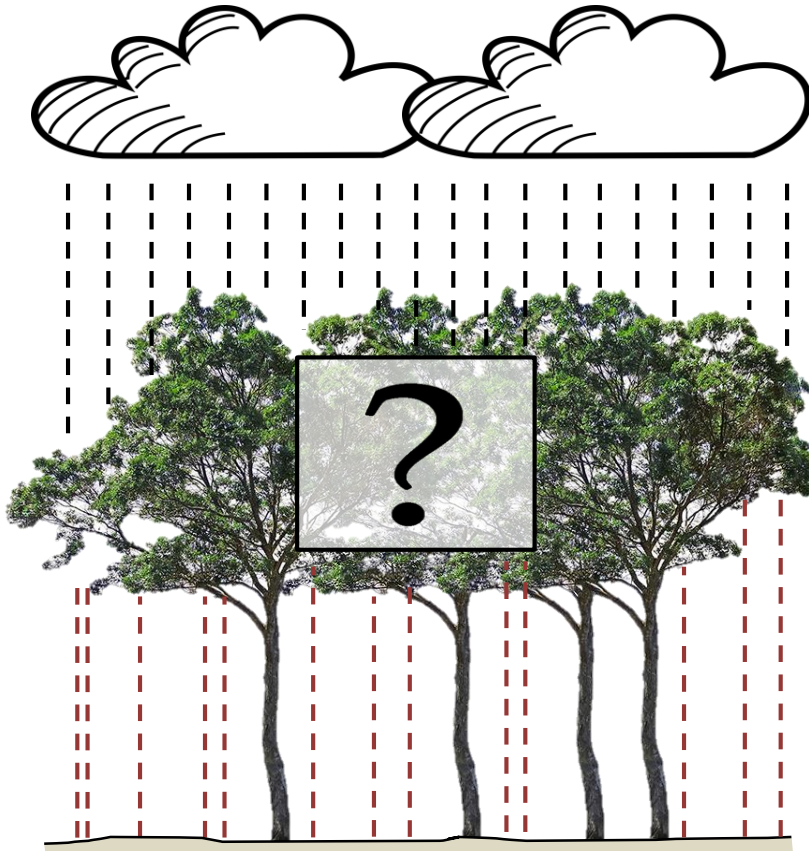
# MODELO DE INTERCEPTAÇÃO (RUTTER MODIFICADO)



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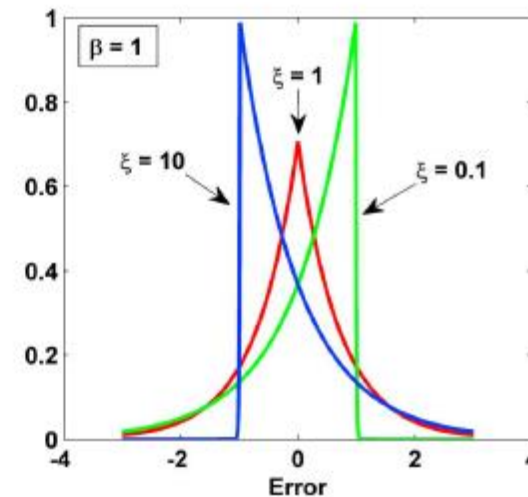
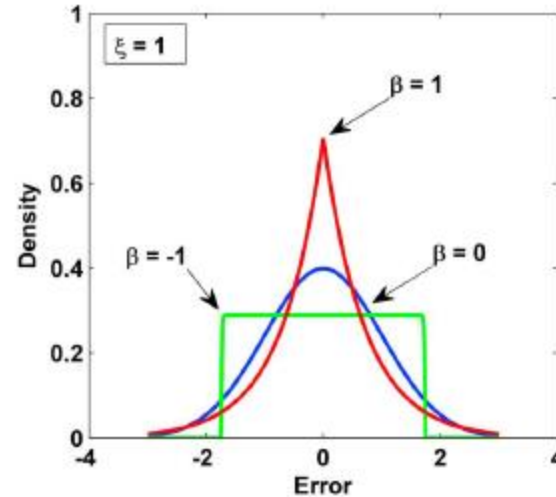


4 parâmetros

## GENERALIZED LIKELIHOOD FUNCTION

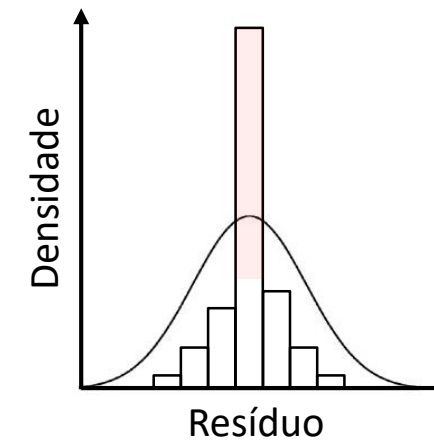
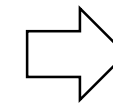
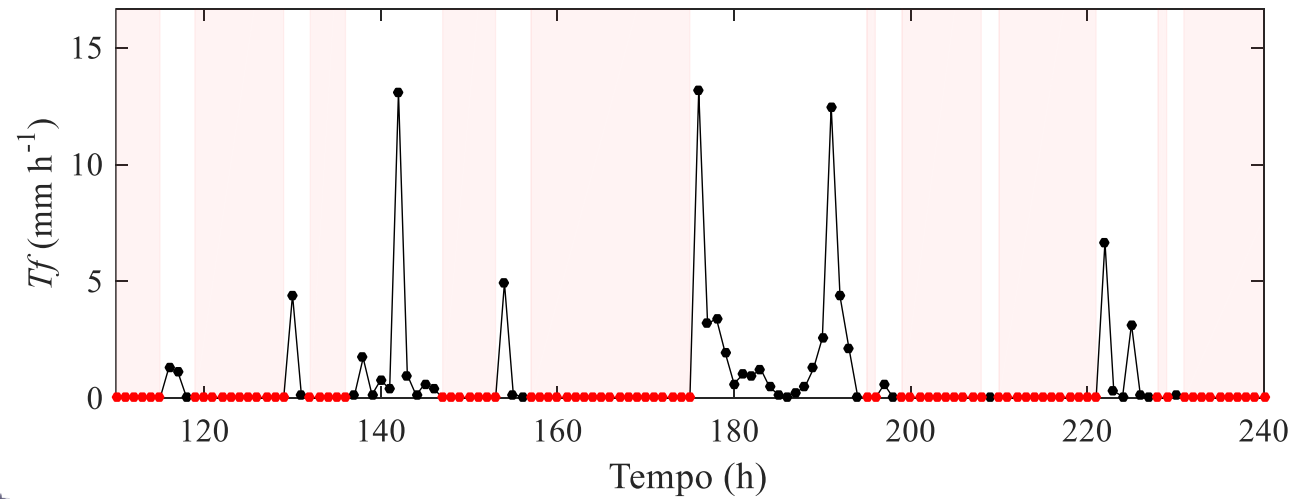
(Schoups & Vrugt, 2010)

$$\ell = n \log \frac{2\sigma_\xi \omega_\beta}{\xi + \xi^{-1}} - \sum_{t=1}^n \log \sigma_t - c_\beta \sum_{t=1}^n |a_{\xi,t}|^{2/(1+\beta)}$$



## GENERALIZED LIKELIHOOD FUNCTION (Schoups & Vrugt, 2010)

$$\ell = n \log \frac{2\sigma_\xi \omega_\beta}{\xi + \xi^{-1}} - \sum_{t=1}^n \log \sigma_t - c_\beta \sum_{t=1}^n |a_{\xi,t}|^{2/(1+\beta)}$$





## GENERALIZED LIKELIHOOD FUNCTION

(Schoups & Vrugt, 2010)

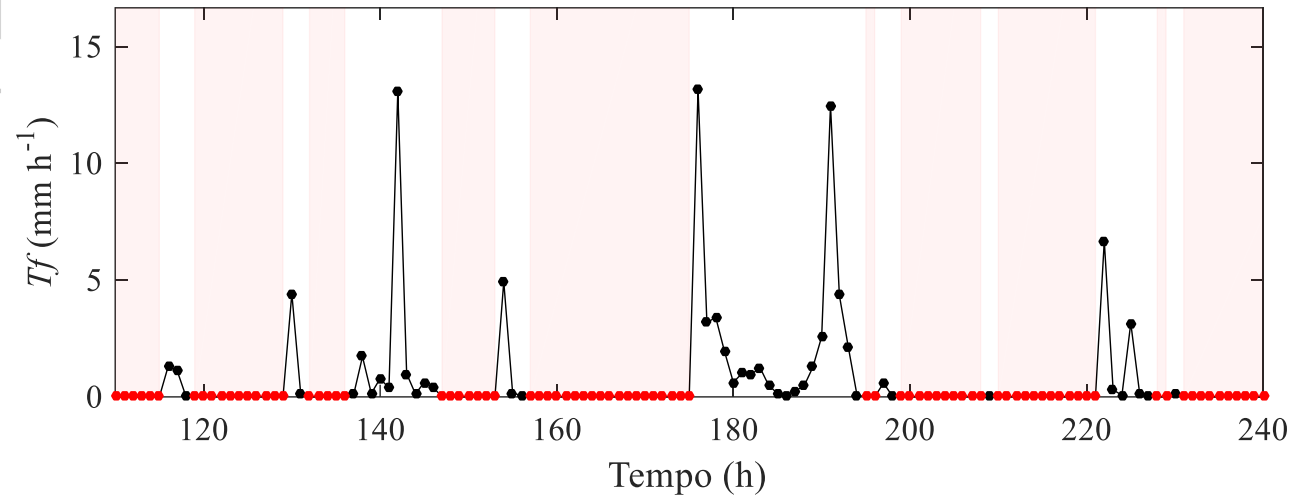
$$\ell = n \log \frac{2\sigma_\xi \omega_\beta}{\xi + \xi^{-1}} - \sum_{t=1}^n \log \sigma_t - c_\beta \sum_{t=1}^n |a_{\xi,t}|^{2/(1+\beta)}$$

## GENERALIZED LIKELIHOOD FUNCTION MODIFICADA

$$\ell = n_1 \log \rho + n_2 \log(1 - \rho) + n_2 \log \frac{2\sigma_\xi \omega_\beta}{\xi + \xi^{-1}} - \sum_{t_2=1}^{n_2} \log \sigma_{t_2} - c_\beta \sum_{t_2=1}^{n_2} |a_{\xi,t_2}|^{2/(1+\beta)} +$$

$$n_3 \log \frac{2\sigma_\xi \omega_\beta}{\xi + \xi^{-1}} - \sum_{t_3=1}^{n_3} \log \sigma_{t_3} - c_\beta \sum_{t_3=1}^{n_3} |a_{\xi,t_3}|^{2/(1+\beta)}$$

## GENERALIZED LIKELIHOOD FUNCTION (Schoups & Vrugt)



## GENERALIZED LIKELIHOOD FUNCTION MODIFICADA

$$\ell = n_1 \log \rho + n_2 \log(1 - \rho) + n_2 \log \frac{2\sigma_\xi \omega_\beta}{\xi + \xi^{-1}} - \sum_{t_2=1}^{n_2} \log \sigma_{t_2} - c_\beta \sum_{t_2=1}^{n_2} |a_{\xi, t_2}|^{2/(1+\beta)} +$$

$$n_3 \log \frac{2\sigma_\xi \omega_\beta}{\xi + \xi^{-1}} - \sum_{t_3=1}^{n_3} \log \sigma_{t_3} - c_\beta \sum_{t_3=1}^{n_3} |a_{\xi, t_3}|^{2/(1+\beta)}$$

## GENERALIZED LIKELIHOOD FUNCTION

(Schoups & Vrugt, 2010)

$$\ell = n \log \frac{2\sigma_\xi \omega_\beta}{\xi + \xi^{-1}} - \sum_{t=1}^n \log \sigma_t - c_\beta \sum_{t=1}^n |a_{\xi,t}|^{2/(1+\beta)}$$

## GENERALIZED LIKELIHOOD FUNCTION MODIFICADA

$$\ell = n_1 \log \rho + n_2 \log(1 - \rho) + n_2 \log \frac{2\sigma_\xi \omega_\beta}{\xi + \xi^{-1}} - \sum_{t_2=1}^{n_2} \log \sigma_{t_2} - c_\beta \sum_{t_2=1}^{n_2} |a_{\xi,t_2}|^{2/(1+\beta)} + n_3 \log \frac{2\sigma_\xi \omega_\beta}{\xi + \xi^{-1}} - \sum_{t_3=1}^{n_3} \log \sigma_{t_3} - c_\beta \sum_{t_3=1}^{n_3} |a_{\xi,t_3}|^{2/(1+\beta)}$$

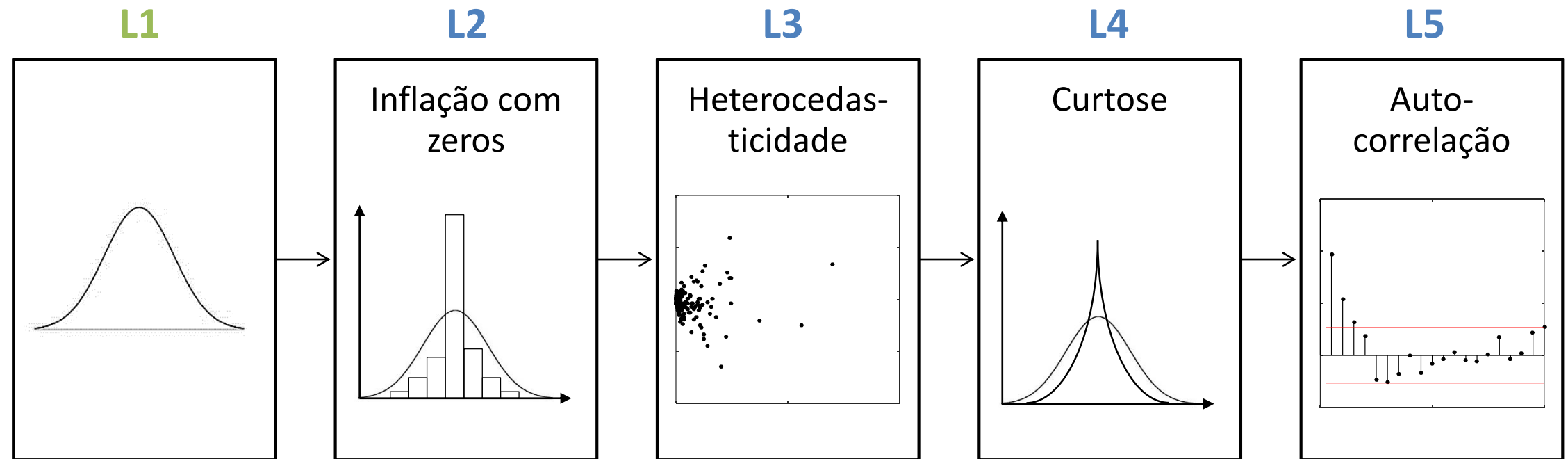
resíduo = 0

resíduo ≠ 0 para observação = 0

resíduo ≠ 0 para observação ≠ 0

## MODELO PARA RESÍDUOS CONSIDERADOS

- **Aumento gradual da complexidade** do modelo para resíduos



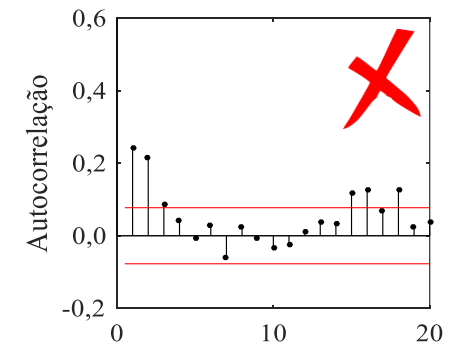
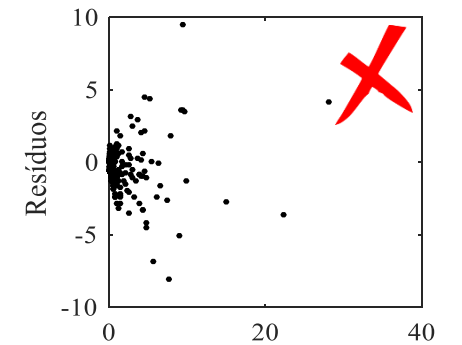
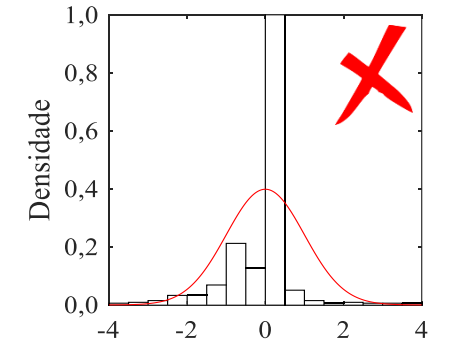
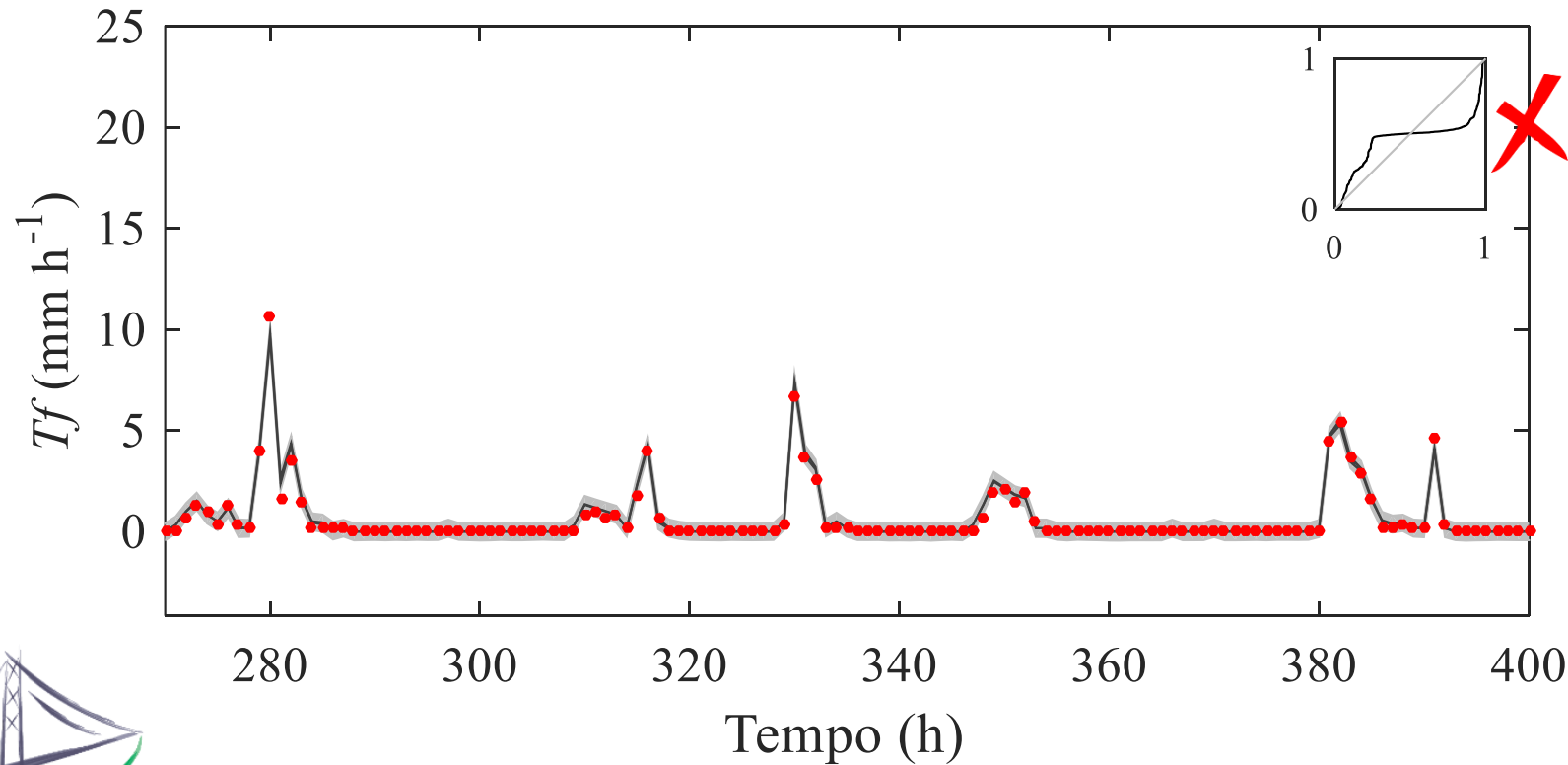
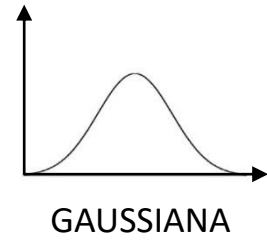
GENERALIZED  
LIKELIHOOD FUNCTION

GENERALIZED LIKELIHOOD  
FUNCTION MODIFICADA

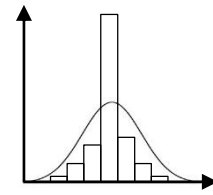
# RESULTADOS



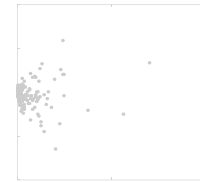
# CALIBRAÇÃO: L1



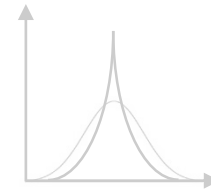
# CALIBRAÇÃO: L2



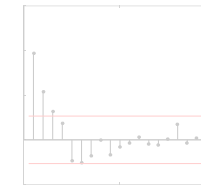
INFLAÇÃO  
COM ZEROS



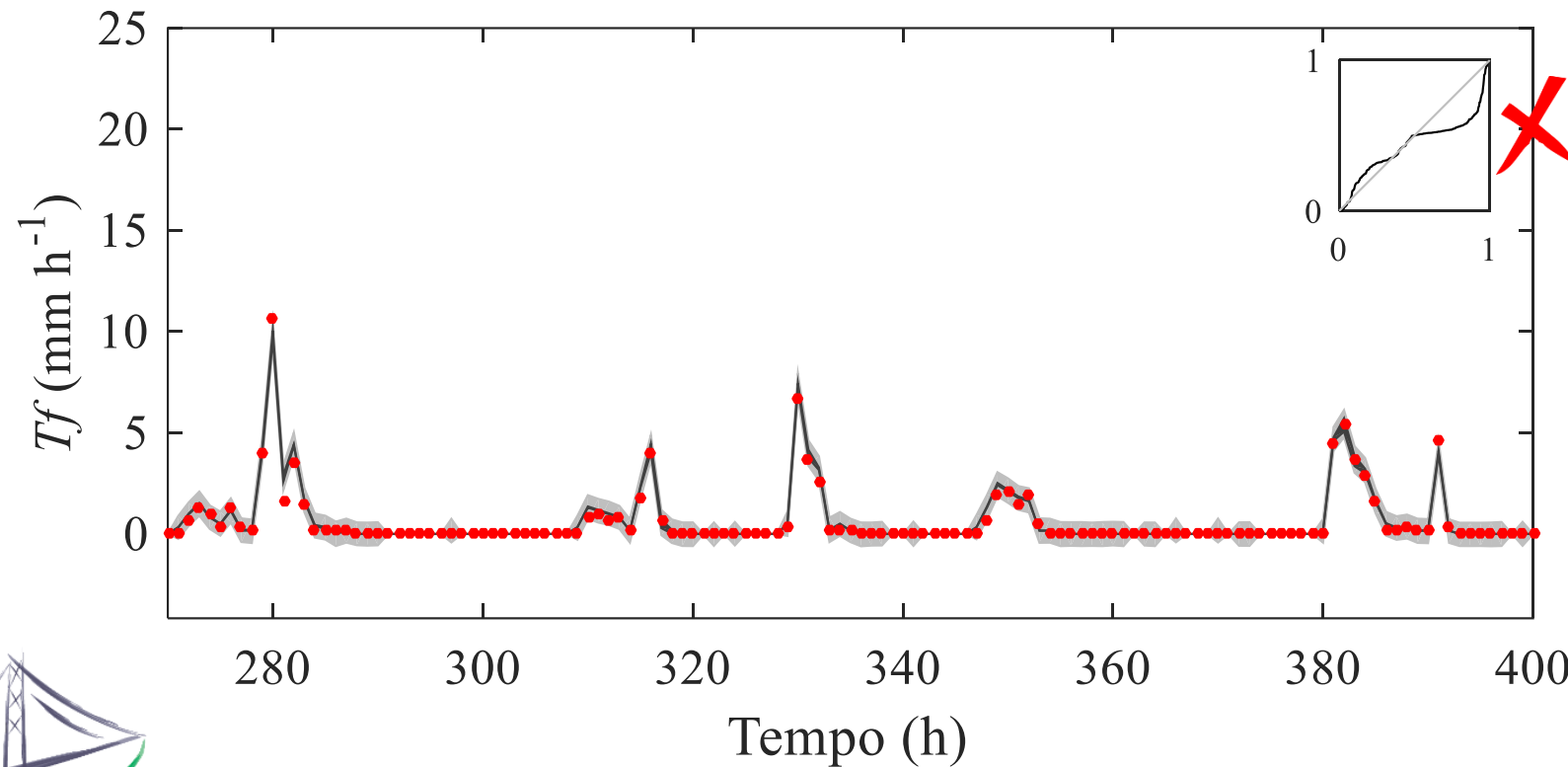
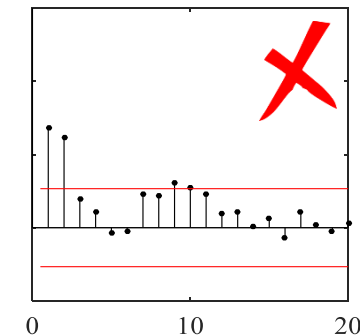
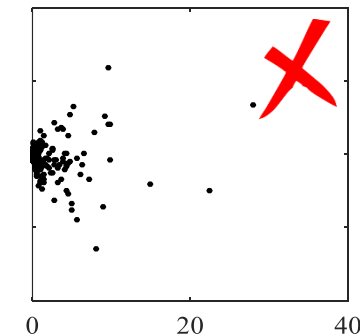
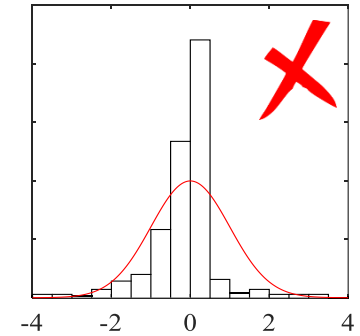
HETEROSCE-  
DASTICIDADE



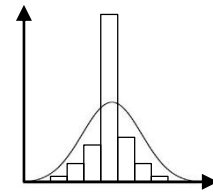
CURTOSE



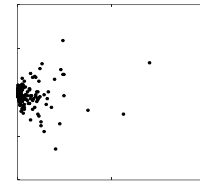
AUTO-  
CORRELAÇÃO



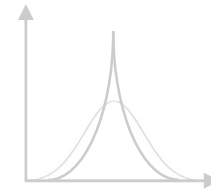
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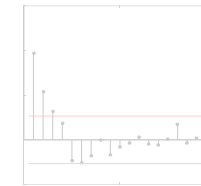
INFLAÇÃO COM ZEROS



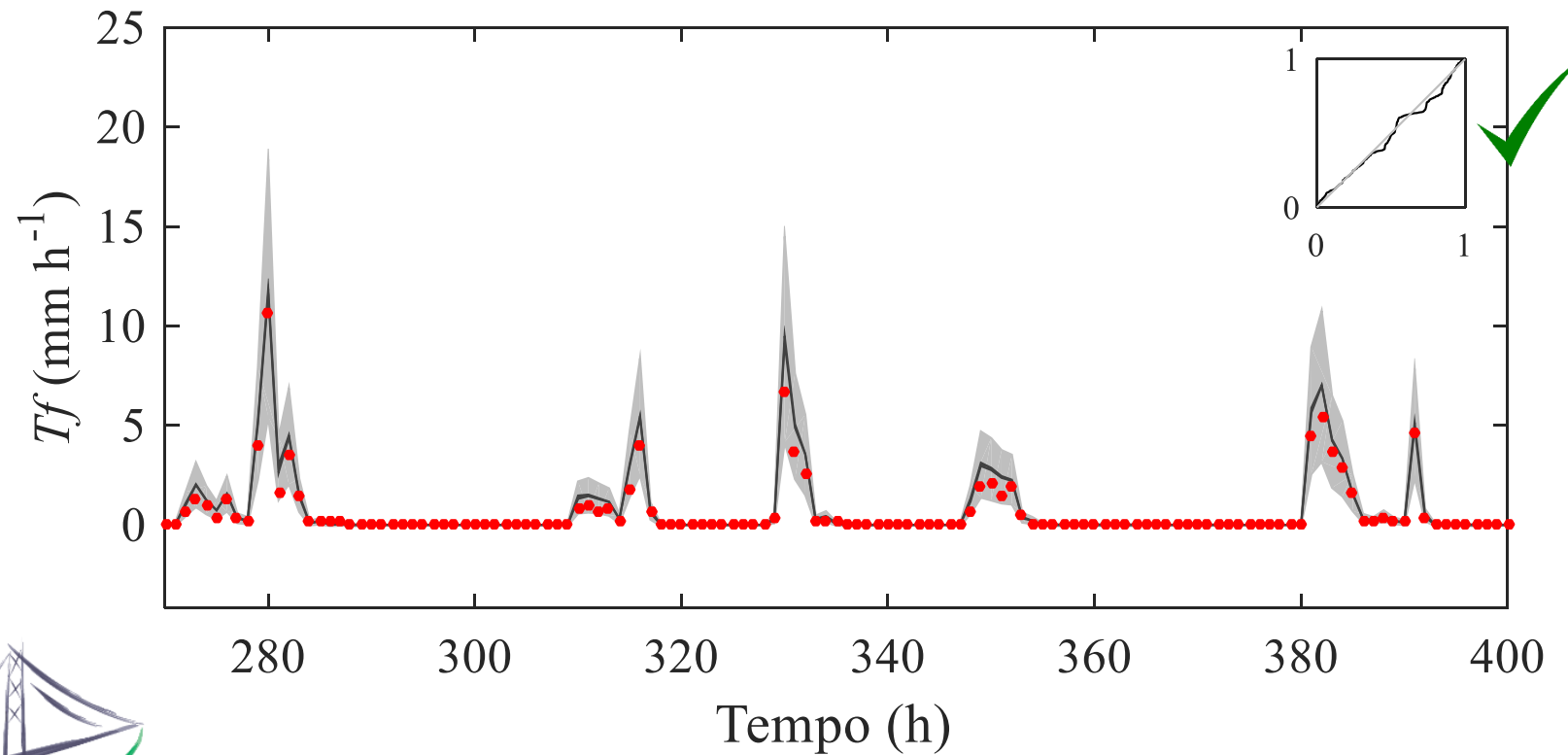
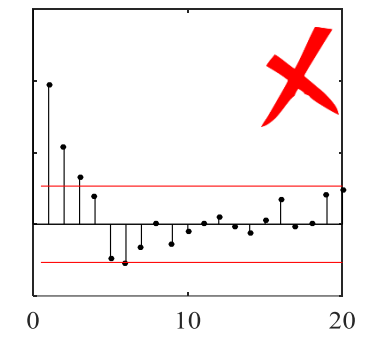
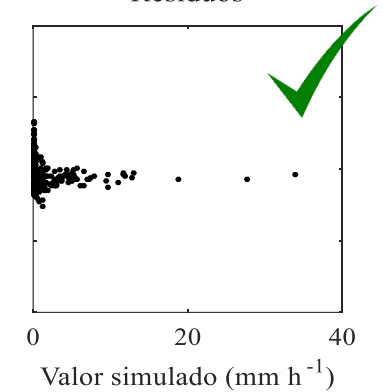
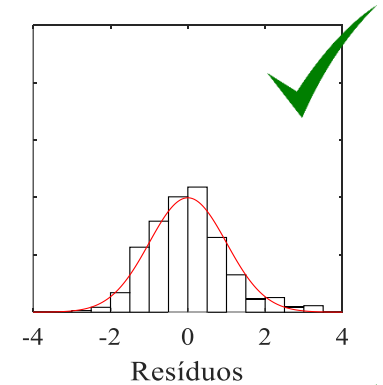
HETEROSCE-DASTICIDADE



CURTOSE

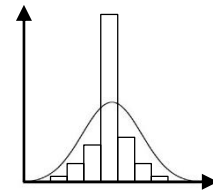


AUTO-CORRELAÇÃO

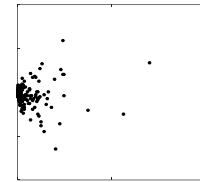




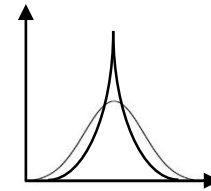
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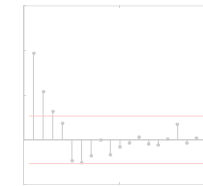
INFLAÇÃO  
COM ZEROS



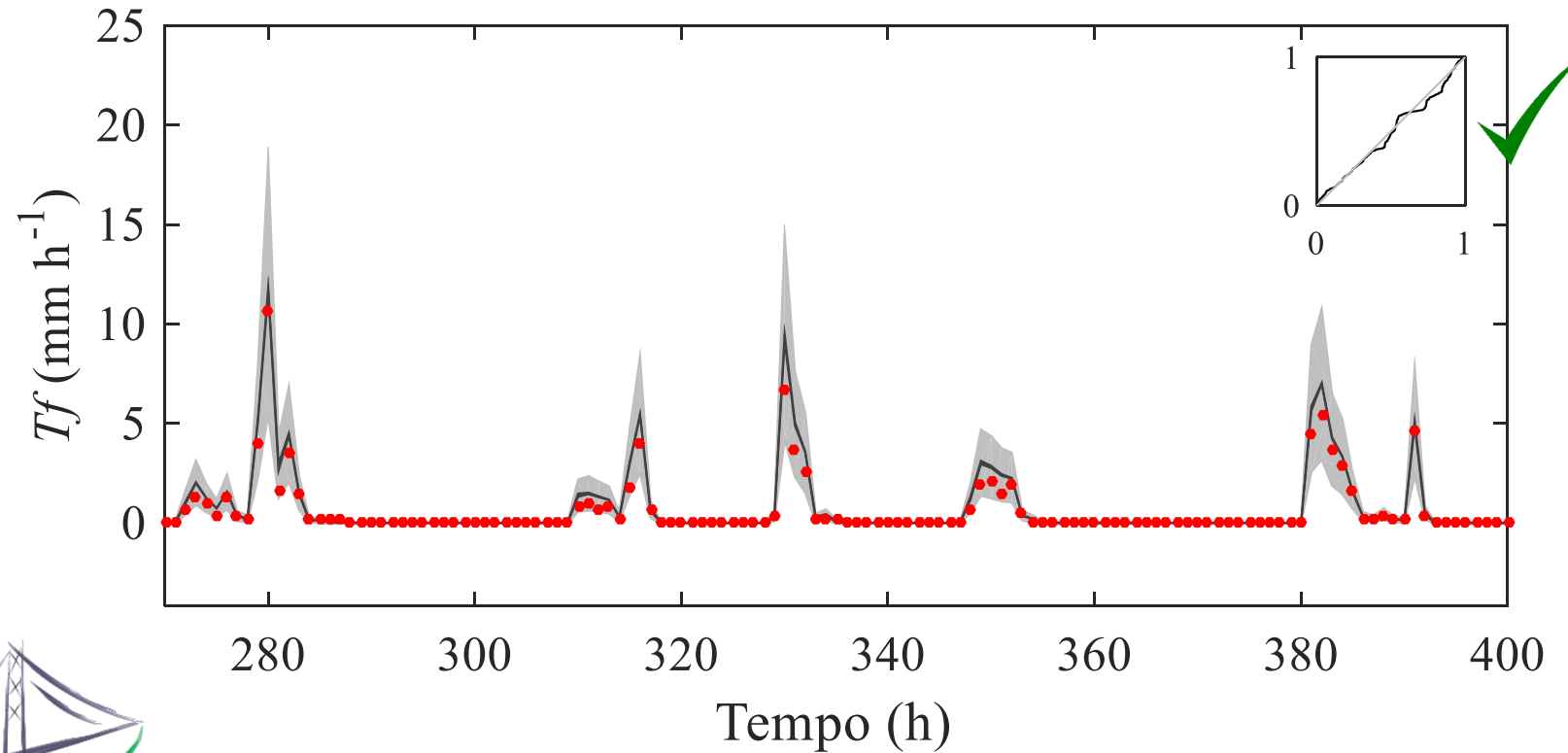
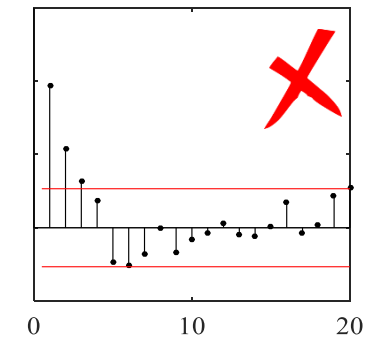
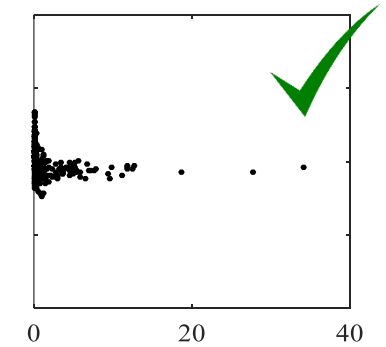
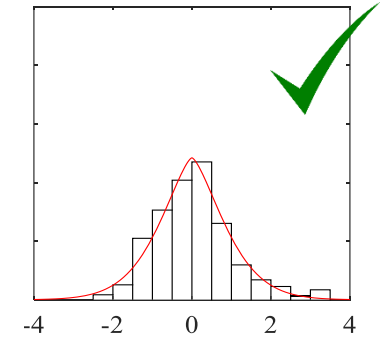
HETEROSCE-  
DASTICIDADE



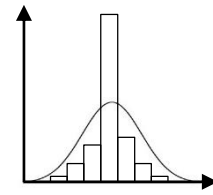
CURTOSE



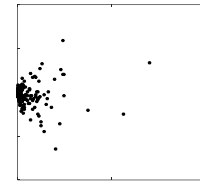
AUTO-  
CORRELAÇÃO



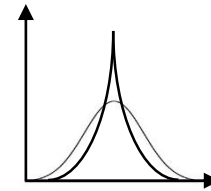
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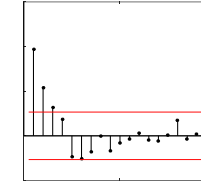
INFLAÇÃO COM ZEROS



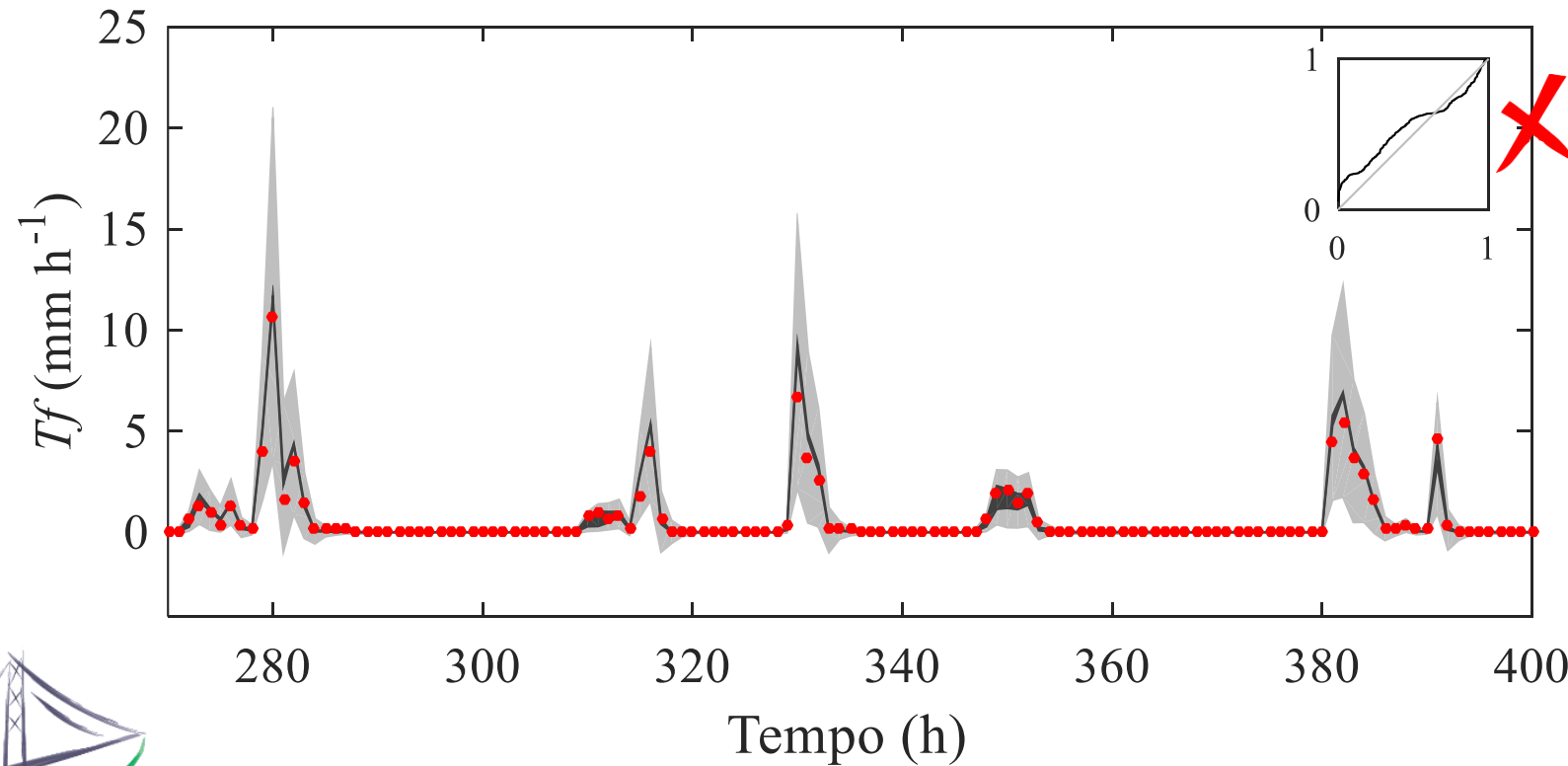
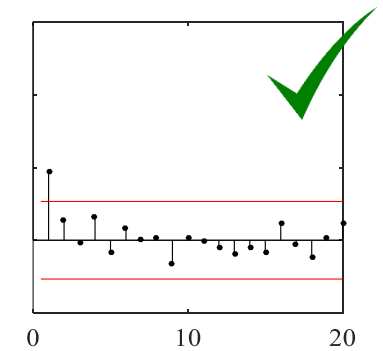
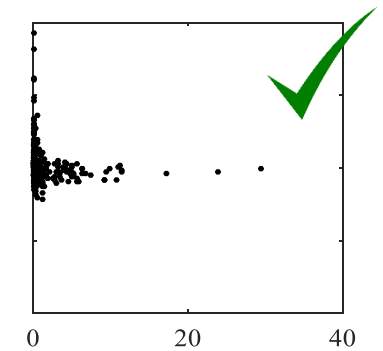
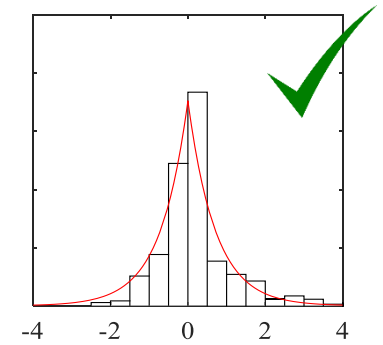
HETEROSCEDASTICIDADE



CURTOSE

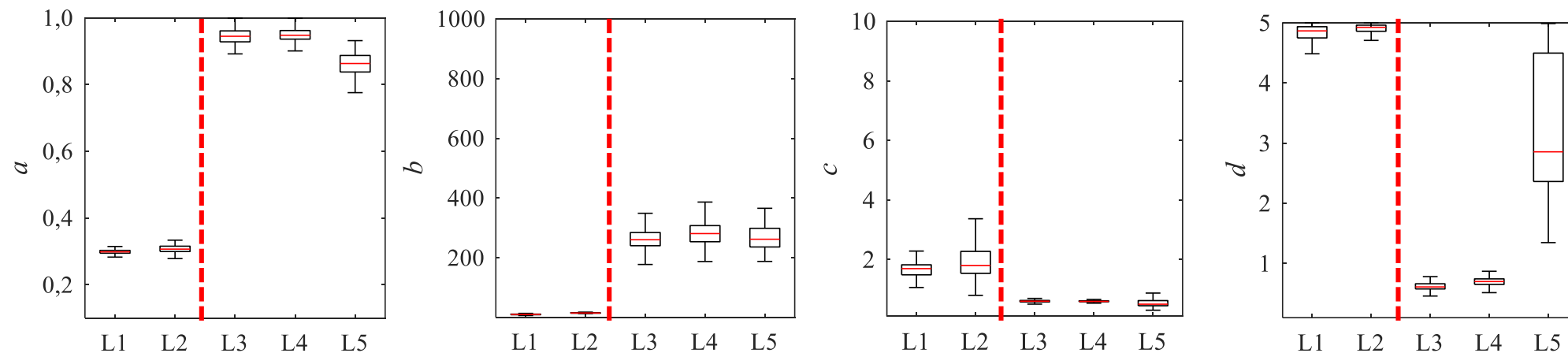


AUTO-CORRELAÇÃO



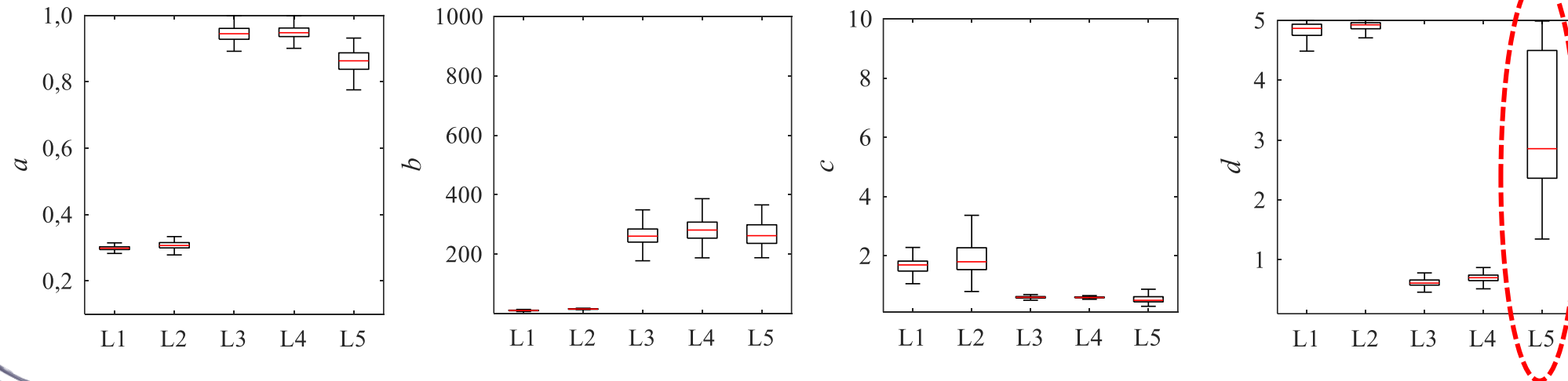
## IMPACTO NA INFERÊNCIA DOS PARÂMETROS

- Consideração da **heteroscedasticidade** dos resíduos é especialmente importante
- Correlação entre parâmetros do modelo autoregressivo com parâmetros do modelo de interceptação

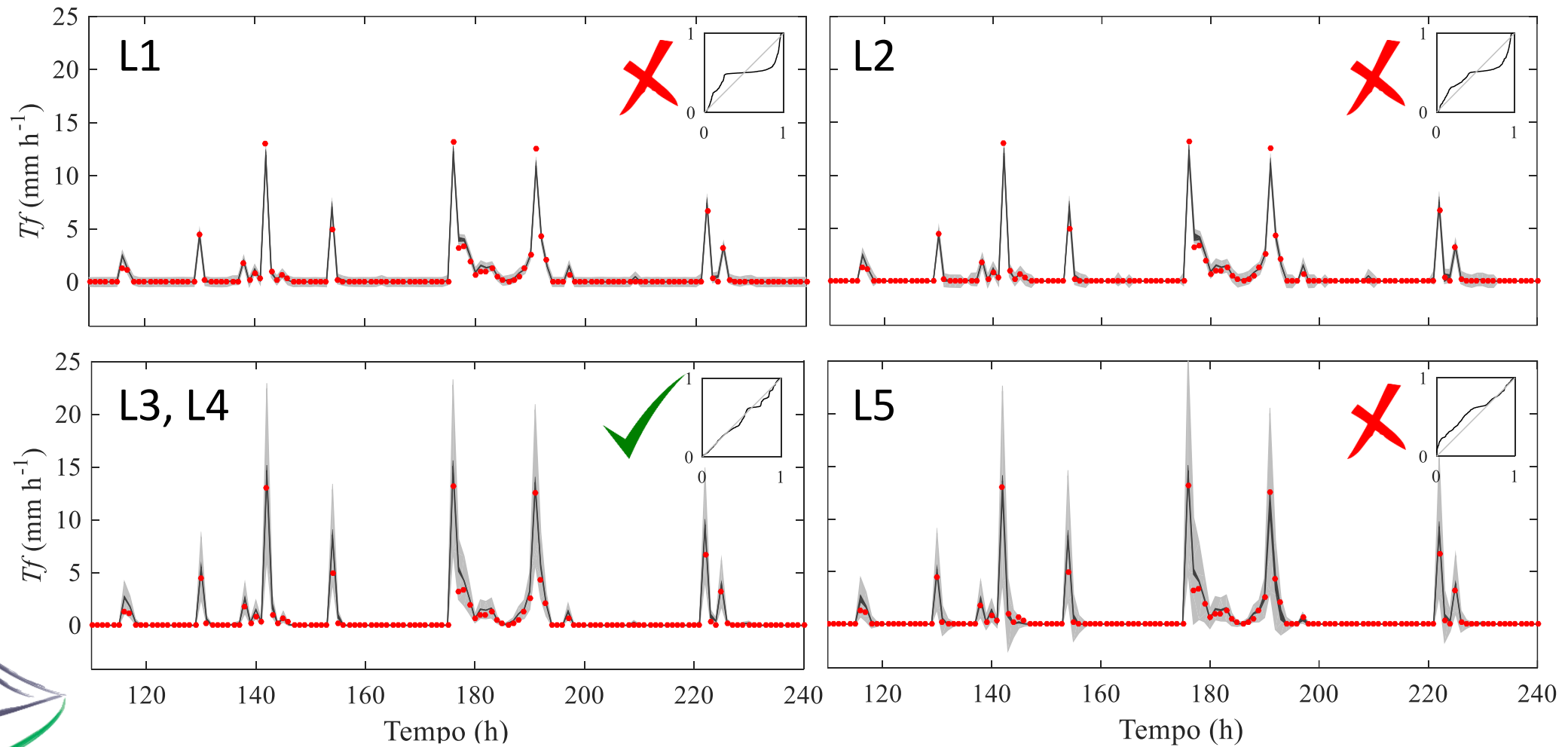


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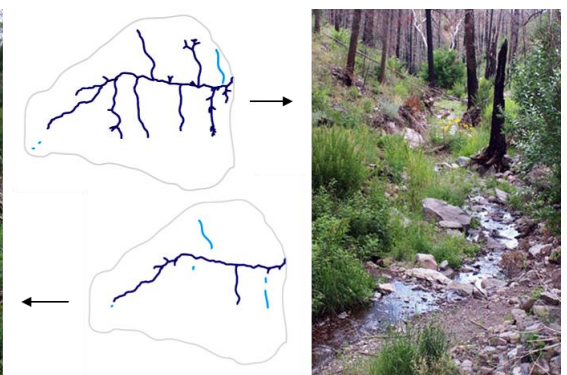
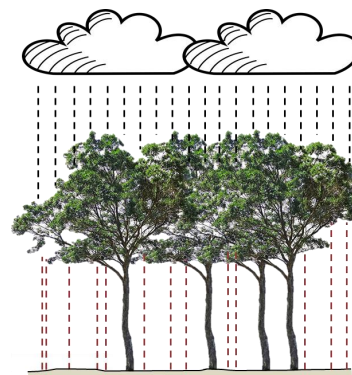
# IMPACTO NA ESTIMATIVA DA INCERTEZA: VALIDAÇÃO



# CONCLUSÃO

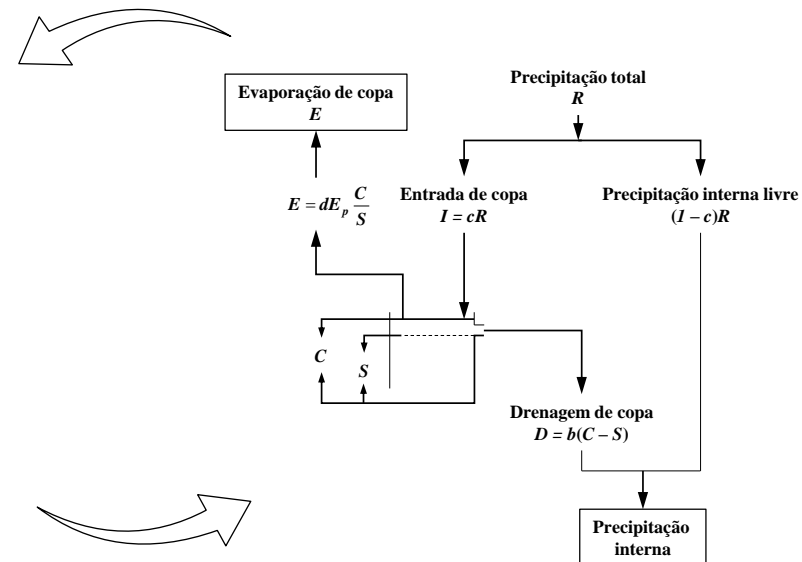
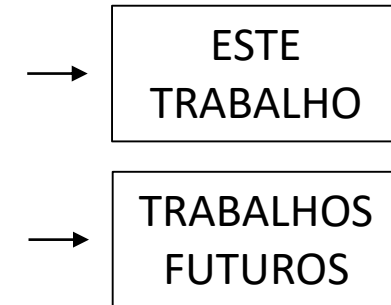


- A correta caracterização dos resíduos do modelo melhora a qualidade da incerteza estimada, transmitindo uma maior confiabilidade aos valores dos parâmetros obtidos.
- O modelo para resíduos apresentado também pode ser utilizado em outros estudos em que a série de dados possui muitas observações iguais a zero.



- Acreditamos que progresso no desenvolvimento de **melhores representações para os processos hidrológicos** pode ser obtido com

1. reconhecimento da incerteza associada aos resultados dos modelos
2. redução da incerteza







**MUITO OBRIGADA!**

**40** anos  
Associação Brasileira  
de Recursos Hídricos



## REFERÊNCIAS

GIGLIO, J.N. **Interceptação da chuva em pequena bacia experimental coberta por Floresta Ombrófila Mista**. 2013. 194 f. Dissertação (Mestrado em Engenharia Ambiental) – Universidade Federal de Santa Catarina, Florianópolis, 2013.

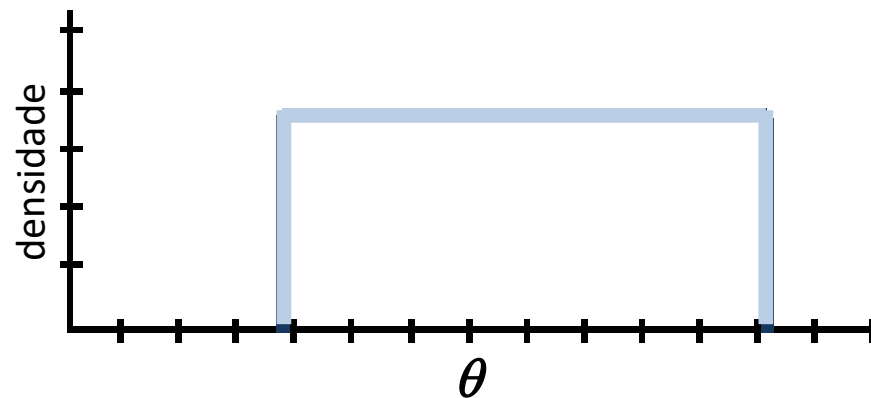
SCHOUPS, G.; VRUGT, J.A. A formal likelihood function for parameter and predictive inference of hydrologic models with correlated, heteroscedastic, and non-Gaussian errors. **Water Resour. Res.**, 46, W10531, 2010.



## DIFFERENTIAL EVOLUTION ADAPTIVE METROPOLIS (DREAM)

- Amostrador do tipo Markov Chain Monte Carlo
- Permite a inferência conjunta dos parâmetros do modelo ( $\theta$ ) e da incerteza

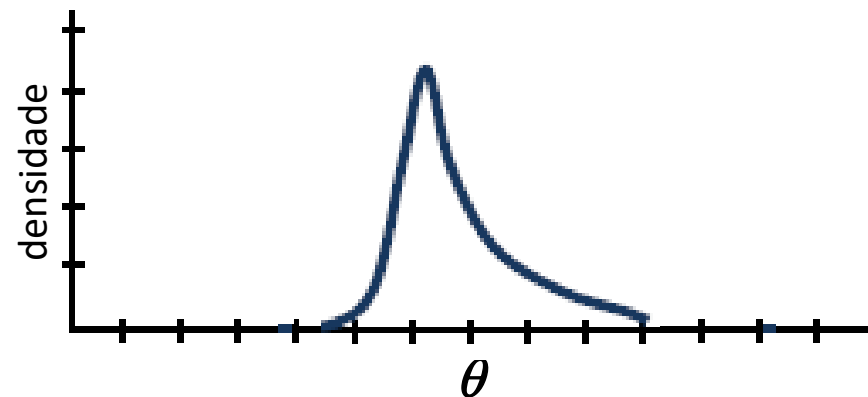
$$\overset{\text{posterior}}{\boxed{p(\theta|\mathbf{D})}} = \frac{p(\mathbf{D}|\theta)p(\theta)}{p(\mathbf{D})} \propto \underset{\substack{\text{função de} \\ \text{verossimilhança}}}{L(\theta|\mathbf{D})} \boxed{p(\theta)} \underset{\text{distribuição} \\ \text{a priori}}{\text{a priori}}$$



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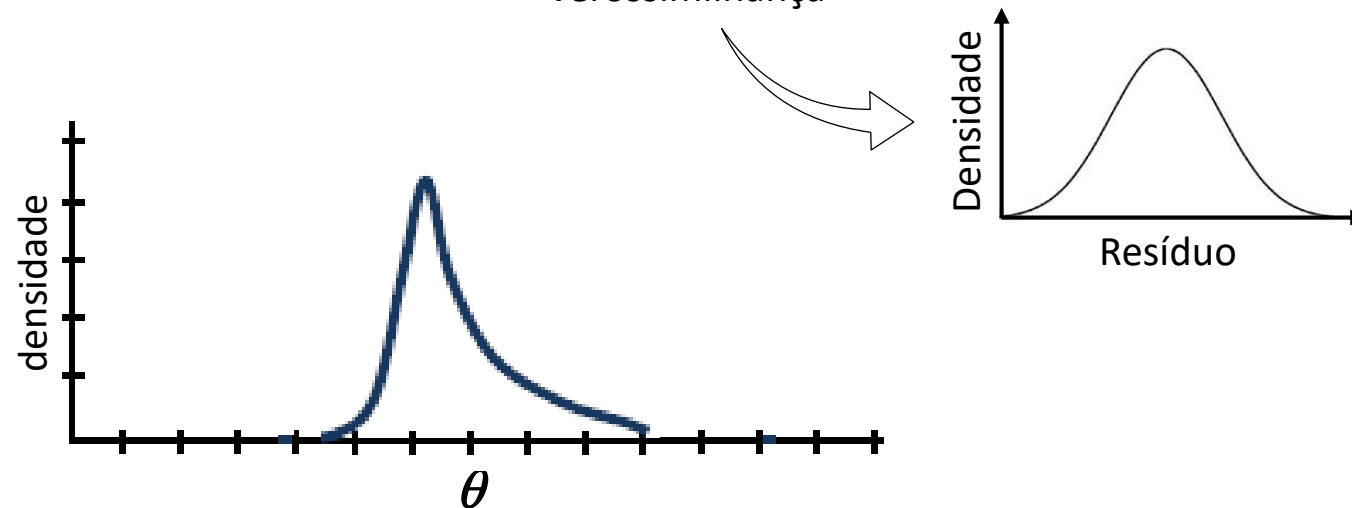


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função de verossimilhança



## MODELO PARA RESÍDUOS CONSIDERADOS

- L1: Erros gaussianos, homoscedásticos e independentes
- L2: Erros gaussianos, homoscedásticos, independentes e **inflados com zero**
- L3: Erros gaussianos, **heteroscedásticos**, independentes e inflados com zero
- L4: Erros **não-gaussianos**, heteroscedásticos, independentes e inflados com zero
- L5: Erros não-gaussianos, heteroscedásticos, **autocorrelacionados** e inflados com zero

